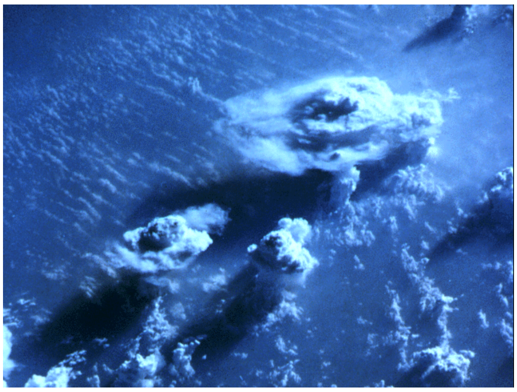
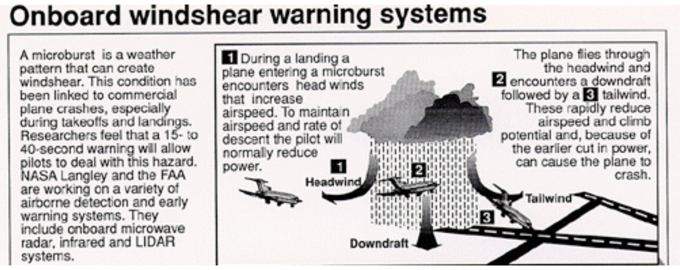
Cumulus Dynamics



Motivation



Different cumulus types

- Fair weather cumulus (Cu): 1 km horizontal and vertical
- Cumulus congestus (towering cumulus): several km width and depth; forms from aggregates of discrete smaller buoyant bubbles within the cloud, that rise one after the other
- Individual cumulonimbus (Cb): thunderstorms
- Mesoscale convective systems (MCS): complexes of thunderstorms

Buoyancy

- All convective clouds owe their existence to air becoming buoyant on a local scale (< 10 km in horizontal extent)
- Thus need to consider buoyancy in the vertical component of the momentum equation:

$$\frac{dw}{dt} = -\frac{1}{\rho_o}\frac{\partial p^*}{\partial z} + B \quad (1)$$

where ρ_o = reference state density; p^* = deviation of the pressure from its reference-state (= environment).

- The buoyancy is given as:

$$B \approx g\left(\frac{T^*}{T_o} - \frac{p^*}{p_o} + 0.61q_v^* - q_H\right) \quad (2)$$

where T^*, q_v^* = deviation of T and water vapor mixing ratio from their reference-states ($T_o, q_{v,o}$); q_H = mixing ratio of hydrometeors

Buoyancy

- In convective clouds all 4 terms are of the same order of magnitude
- A temperature perturbation of 1 K
  = perturbation of 3 hPa in pressure
  = 5 g/kg in water vapor mixing ratio
  = 3 g/kg in hydrometeor mixing ratio
- Thus need to consider pressure perturbations

Pressure perturbation I

- If a parcel in a horizontally uniform atmosphere (same T) has a lower density, its pressure must also be lower than that of the environment.
- The horizontal pressure gradient accelerates environmental air toward the base of the buoyant parcel.
- This air also replaces the upward moving air of the parcel
- Mathematical description of this phenomenon by combining the horizontal and vertical equations of motion with the continuity equation neglecting friction and the Coriolis force

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_o}\nabla p^* + B\vec{k} - \vec{v}\cdot\nabla\vec{v} \quad (3)$$

Pressure perturbation II

- $$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_o}\nabla p^* + B\vec{k} - \vec{v}\cdot\nabla\vec{v} \quad (4)$$
- Multiply by ρ_o and take 3D divergence:
$$\frac{\partial}{\partial t}(\nabla\cdot\rho_o\vec{v}) = -\nabla^2 p^* + \frac{\partial}{\partial z}(\rho_o B) - \nabla\cdot(\rho_o\vec{v}\cdot\nabla\vec{v}) \quad (5)$$
- Assuming that we can neglect the time derivative of the density (anelastic system that eliminates sound waves) means that the left side is 0 so that
$$\nabla^2 p^* = \nabla^2(p_B^* + p_D^*) = F_B + F_D = \frac{\partial}{\partial z}(\rho_o B) - \nabla\cdot(\rho_o\vec{v}\cdot\nabla\vec{v}) \quad (6)$$
- I.e. the pressure perturbations arise from a buoyancy source (F_B) and a dynamic source (F_D) → to come later.

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Buoyancy source of pressure perturbation I

- Is analogous to Poisson's equation in electrostatics
$$\nabla^2 p_B^* = F_B \quad (7)$$
- with this solution:

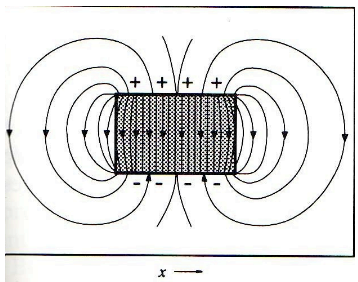

Figure: 7.1 [Houze, 1993]: Vector field of buoyancy pressure-gradient force for a uniformly buoyant parcel. $+, - =$ sign of $-F_B$

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Buoyancy source of pressure perturbation II

- in Fig. 7.1: $-F_B > 0$, the buoyancy pressure-gradient acceleration (BPGA) field diverges and for $-F_B < 0$ converges.
- Everywhere except at the top and bottom of the parcel, $F_B = 0$.
- The lines of the BPGA file are shown as streamlines.
- The BPGA field diverges at the parcel top and converges at its bottom.
- Outside the parcel, lines indicate the directions of force acting to produce the compensating motions in the environment that are required to satisfy mass continuity when the buoyant parcel moves upward.

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Buoyancy source of pressure perturbation III

- Within the parcel, the BPGA field is downward indicating that the upward acceleration of buoyancy is partly counteracted by a downward BPGA.
- This counteraction must occur because some of the buoyancy of the parcel has to be used to move environmental air out of the way in order to preserve mass continuity while the parcel rises
- I.e. for a given amount of buoyancy, the narrower the parcel, the larger the upward acceleration
- In Cu and Cb, BPGA is especially important near the tops of growing clouds, where rising towers are actively pushing environmental air out of the way

Entrainment: general considerations

- Entrainment: incorporation of environmental air into the cloud
- The intensity of turbulence inside convective clouds is much greater than in the surrounding environment because both strong shear and buoyancy generation exist.
- Some of the internal cloud motions are organized overturning and rotation on the scale of the cloud itself, while some turbulence is of a smaller scale and more random
- Mixing occurs across all the edges of the cloud, as a result of both the random and the organized motions.
- As a result of these motions and to satisfy mass continuity, the cloud becomes diluted by entrainment.

Continuous, homogeneous entrainment

- Historical, simplified approach that treats entrainment as continuous in time and uniform in space.
- Consider quantity A (energy, mass or momentum per unit mass of air)
- In-cloud value: A_c, environmental value A_e
- Change of A_c with time:

$$\frac{dA_c}{dt} = \left(\frac{dA_c}{dt}\right)_S + \underbrace{\frac{1}{m}\left(\frac{dm}{dt}\right)_\epsilon}_{\Lambda_t}(A_e - A_c) \tag{8}$$

where $(dA_c/dt)_S$ refers to the rate of change of A_c even without the parcel exchanging mass with the environment and Λ_t is the temporal entrainment (ϵ)

Entrainment in a rising cloud element

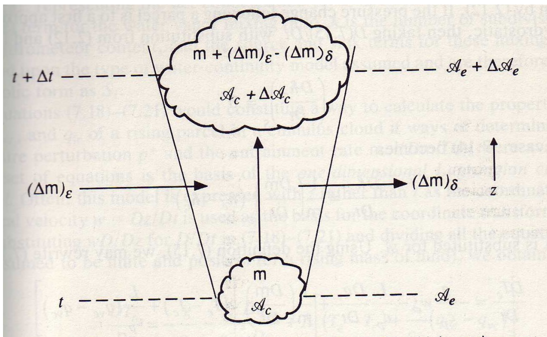


Figure 7.3 Idealization of a rising cloud element interacting with its environment.

Figure: Houze, 1993

Modification of temperature inside the cloud

- Start from the modified first law of thermodynamics:

$$\frac{dT_c}{dt} = \underbrace{-\frac{g}{c_p}w_c}_{\text{dry-adiab. cooling;}} \underbrace{-\frac{L}{c_p}\frac{dq_{vc}}{dt}}_{\text{latent heating}} + \underbrace{\Lambda_t\left[(T_e - T_c) + \frac{L}{c_p}(q_{ve} - q_{vc})\right]}_{\text{effects of entrainment}} \quad (9)$$

- Obtain T change with altitude by substituting d/dt with wd/dz and division by w (assumed to be finite and positive for a rising parcel)

$$\frac{dT_c}{dz} = -\frac{g}{c_p} - \frac{L}{c_p}\frac{dq_{vc}}{dz} + \Lambda_z\left[(T_e - T_c) + \frac{L}{c_p}(q_{ve} - q_{vc})\right] \quad (10)$$

where $\Lambda_z = \frac{1}{m}\left(\frac{dm}{dz}\right)_\epsilon$

- This equation explains why observed T ascends inside cloud lie between the moist-adiabatic and the environmental lapse rate.

Entrainment effect on vertical velocity

- Replace A_c with vertical velocity w_c.
- Source term $(dw_c/dt)_S$ is given by equation (1) yielding:

$$\frac{dw_c}{dt} = -\frac{1}{\rho_e}\frac{\partial p^*}{\partial z} + B - \Lambda_t w_c \quad (11)$$

here neglect vertical velocity in the environment which is small as compared to the vertical velocity in the cloud and associate the reference state with the environment ($\rho_o = \rho_e$).

- Change of w_c with height:

$$w_c\frac{dw_c}{dz} = -\frac{1}{\rho_e}\frac{\partial p^*}{\partial z} + B - \Lambda_z w_c^2 \quad (12)$$

2 models of convection

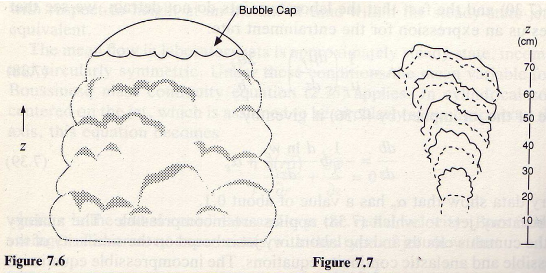


Figure: Bubble model (left) and thermals (right) [7.6 & 7.7; Houze, 1993]

Entrainment

- It was found that buoyant bubbles entrain air; called thermals
- Vertical component of the momentum equation in the presence of entrainment:

$$w_c \frac{dw_c}{dz} = -D_r + B - \frac{0.6}{b} w_c^2 \tag{13}$$

where $D_r = -0.33B$ = parameterization of the vertical pressure-gradient acceleration term.

$$b = \alpha_\epsilon z \tag{14}$$

where z = height of the center of the thermal and $\alpha_\epsilon = 0.2$

Internal circulation in a rising cumulus element

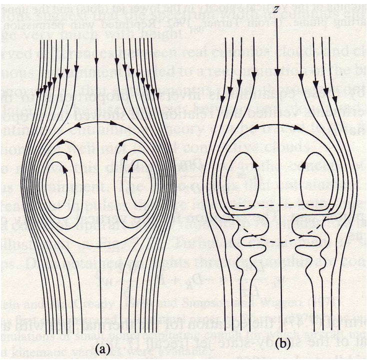


Figure: (a) Hill's vortex (= rising smoke ring); (b) Hill's vortex superposed with a turbulent wake [7.9; Houze, 1993]

- Upward circulation in 7.9b in the center of the element is the mechanism of entrainment
- Upward influx into the element is assumed to come from the wake and thus is composed of an arbitrary mixture of environmental and undiluted cloud air

Discontinuous, inhomogeneous entrainment

- Need for discontinuous, inhomogeneous entrainment arises because continuous, homogeneous entrainment is unable to predict accurate hydrometeor content and cloud height simultaneously nor to reproduce the observed droplet size spectra.
- New idea: entrainment occurs in pulses that are intermittent in both space and time.

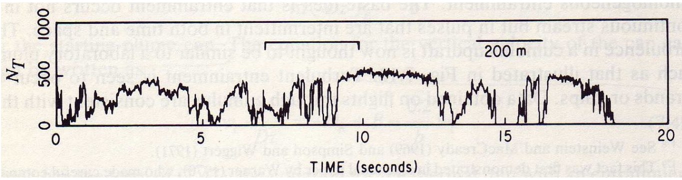


Figure: 7.13 [Houze, 1993]

Turbulent wake (Figure 7.12; Houze, 1993)

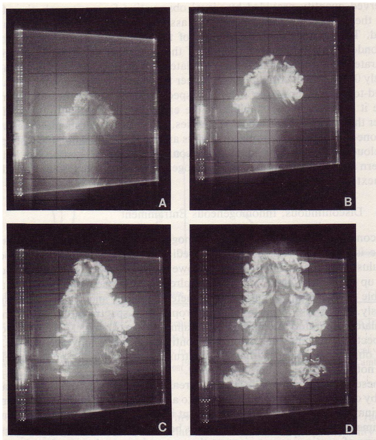


Figure 7.12 Turbulence in a laboratory plume. (Photos courtesy of Shenqyang Shy.)

Discontinuous, inhomogeneous entrainment

- Mixing is slow enough that it remains localized to the region of entrained air
- Thus degree of mixing is spatially inhomogeneous:

$$\tau_e \ll \tau_m \qquad (15)$$

- i.e. time scale of evaporation (τ_e) is much shorter than the time scale of mixing (τ_m) so that drops evaporate completely in ingested blobs, while in other areas drop remain unaffected.
- The opposite would be homogeneous mixing where $\tau_m \ll \tau_e$ so that all cloud droplets shrink

Paluch diagram

- Consider 2 conserved properties F and G. If they are mixed, we obtain:

$$F = (1-f)F_1 + fF_2 \; ; \; G = (1-f)G_1 + fG_2 \qquad (16)$$

  where f fraction of unit mass of the final mixture constituted by fluid originally contained in parcel 2.
- $F(G)$ is a straight line with slope $(F_2 - F_1)/(G_2 - G_1)$
- For warm cloud elements with no precipitation, use the total water mixing ratio $q_T (= q_v + q_c)$ and the equivalent potential temperature Θ_e.
- Need to plot observations from a vertical sounding in the environment and measurements from an aircraft penetration across the cloud at a given altitude.

Paluch diagram

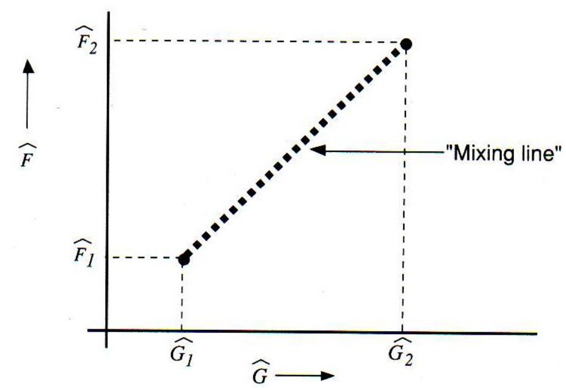
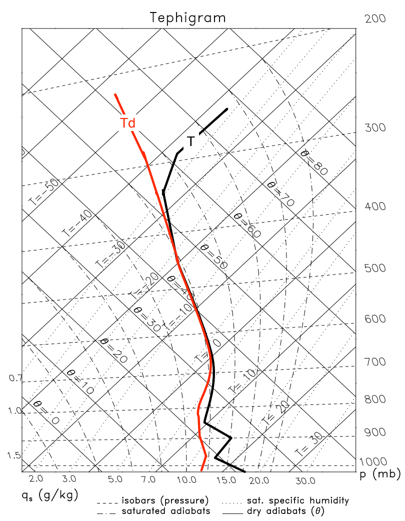


Figure: 7.14 [Houze, 1993]



Exercise:

- Obtain Θ_e and q_t at 900, 650, 600 and 300 hPa from the tephigram
- Sketch them on a Θ_e vs. q_t diagram as depicted on the next slide

Mixing of air from cloud base and cloud top

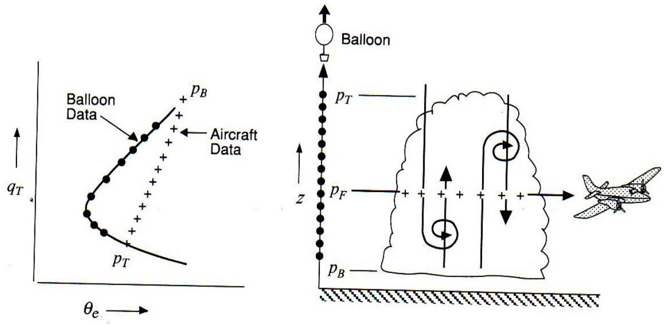


Figure: 7.15 [Houze, 1993]: Example of vertical entrainment and mixing as the dominant mixing processes.

Only some lucky parcels

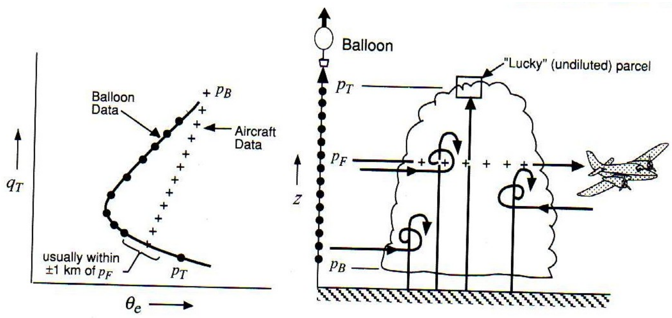


Figure: 7.17 [Houze, 1993]

Only some lucky parcels

- Parcels in figure 7.17 mix in various fractions
- Parcels with entrained environmental air from near flight level, lose their buoyancy, and come to rest near this level.
- Some parcels come to rest at other levels.
- Some parcels never entrained air. These undiluted parcels determine the height of the cloud top.
- This concept allows the average liquid water content at a particular level in a cloud to be far from undiluted even though the cloud top is high, because cloud-top height is determined by the maximal ascent of only the least diluted parcels

Only some lucky parcels

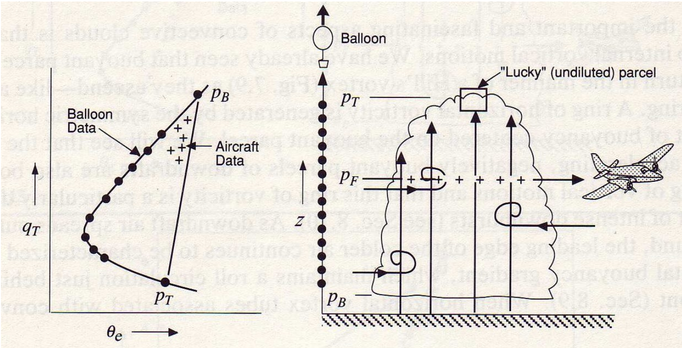


Figure: 7.19 [Houze, 1993]

Collection of parcels

- In reality, parcels could undergo more than one mixing event.
- This, however, can be well approximated by a collection of parcels mixing with entrained air at only one level each
- Equal parcels are released from cloud base to several discrete levels above.
- Upon reaching its designated level, each parcel is split into several subparcels, each mixing with a different fraction of environmental air.
- Each subparcel than rises or sinks to its level of neutral buoyancy, where it is detrained to the environment.

Collection of parcels

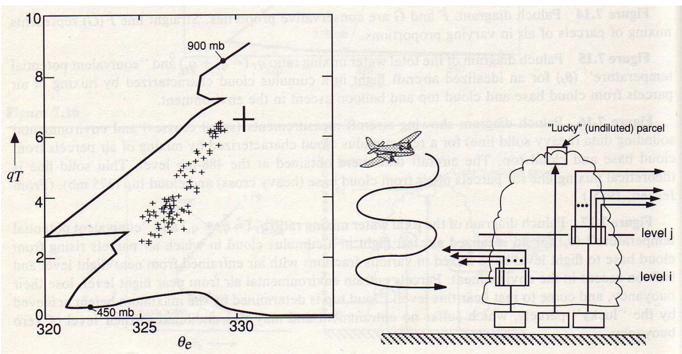


Figure: 7.20&7.21 [Houze, 1993]

Horizontal vorticity I

- Definition of vorticity:

$$\vec{\omega} \equiv \nabla \times \vec{v} = \eta\vec{i} + \xi\vec{j} + \zeta\vec{k} \qquad (17)$$

- Generation of horizontal vorticity ξ about a horizontal axis can develop only through a horizontal gradient of buoyancy B_x (i. e. by baroclinic generation)
- In case of a positively buoyant rising element that overturns like a Hill's vortex (Fig. 7.22a), a maximum of positive buoyancy is centered in the element
- Thus B_x is of equal magnitude and opposite sign on either side of the center line of the element producing counter-rotating vortices on either side of the cloud
- These vortices are entirely consistent with the buoyancy pressure gradient force field in Fig. 7.1.

Generation of horizontal vorticity by horizontal buoyancy gradients

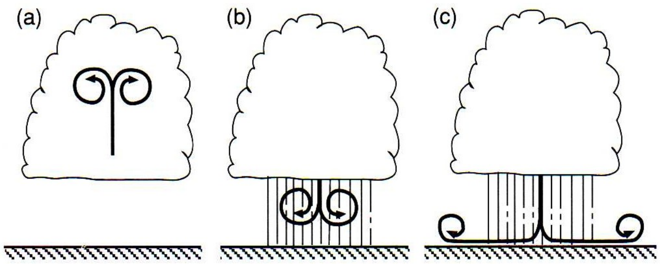


Figure: 7.22 [Houze, 1993]

Horizontal vorticity II

- Negatively buoyant downdrafts associated with evaporative cooling and precipitation drag in the rain shower produce an upside-down version of the overturning updraft (Fig. 7.22b)
- A maximum of negative buoyancy is centered in the element so that B_x is again of equal magnitude and opposite sign of the element and counter-rotating vortices are again produced.
- As the downdraft of dense air spreads out along the ground, a strong buoyancy gradient and vortex is maintained at the leading edge of the outflow (Fig. 7.22c)

Vertical vorticity

- Vorticity about a vertical axis ζ has its origin as horizontal vorticity.
- For instance, convert horizontal vorticity of the environment to vertical vorticity in the cloud (Fig 7.23a).
- Start with horizontal vorticity $\xi = \partial u / \partial z$
- When the updraft of the convective cloud is superimposed on the vortex tube, the tube is deformed upward such that there then exists vorticity around vertical axes in the form of counter-rotation vortices on either side of the updraft core

Conversion of environmental horizontal vorticity to vertical vorticity in cloud

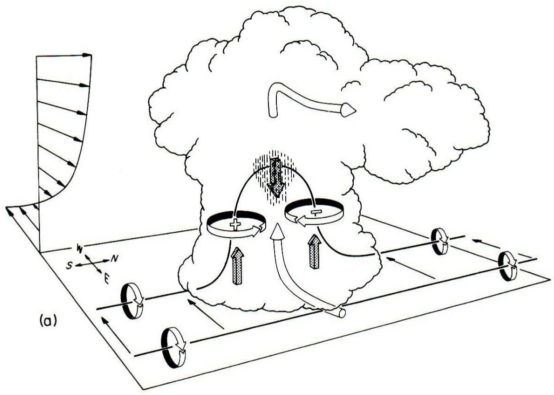


Figure: 7.23 [Houze, 1993]