Instabilities/Microphysics

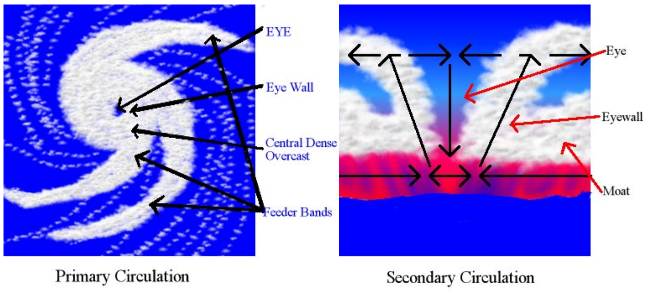


Figure: Primary vs. secondary circulation in a hurricane

Primary vs. secondary circulation in a hurricane

- **Primary circulation**: Tangential flow rotating about the central axis.
- **Secondary circulation** or "in-up-out circulation" (low and middle level inflow, upper-level outflow and the circulation in the eye).
- Need to combine these two components for air parcels spiralling inwards, upwards and outwards.
- The primary circulation is approx. in gradient wind balance.
- If it were in exact balance, there wouldn't be any secondary circulation
- Convection, friction and moist thermodynamics are required to maintain the secondary circulation.

Parameters describing hurricane formation

(Gray, MAP, 1998)

- Coriolis parameter (f)
- Low-level relative vorticity (ζ)
- Inverse of the tropospheric vertical wind shear ($1/S_z$)
- Ocean thermal energy (E)
- The difference in equivalent potential temperature between the surface and 500 hPa ($\Delta\Theta_e$)
- Relative humidity in the mid-troposphere (RH)
- The product of the first 3 parameters specifies a dynamic potential ($f\,\zeta/S_z$)
- The product of the last 3 parameters specifies a thermal potential ($E\,\Delta\Theta_e$ RH)

Scales of atmospheric motion with the phenomena's average size and life span

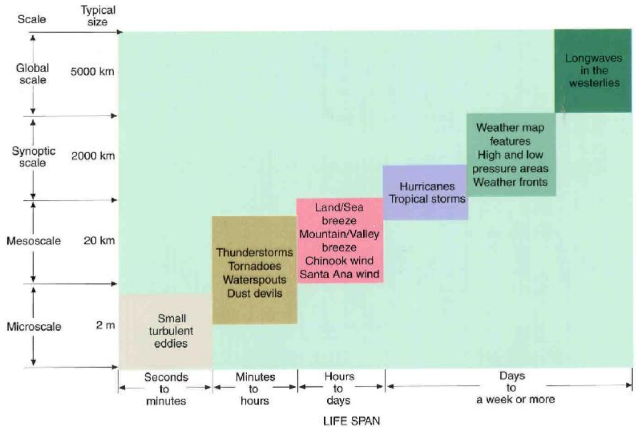


Figure: *www.eas.slu.edu/People/CEGraves/Eas107/scaleofmotion2a.jpg*

Instabilities - general aspects

- Are generally caused by heating of the surface or by cooling aloft
- First dry turbulence will adjust temperature and moisture profile back to neutral conditions
- If turbulence/small scale motions are not sufficient, then organized convection sets in
- For a hurricane, it is required that the atmosphere is potentially unstable to moist convection → to maintain secondary circulation

Perturbation equation

- Rewrite the Navier-Stokes equation in terms of deviations (*) from hydrostatic balance (subscript o):

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho_o}\nabla p^* - f\vec{k}\times\vec{v} - B\vec{k} + \vec{F} \qquad (1)$$

- Where B is the buoyancy, defined as:

$$B = -g\frac{\rho^*}{\rho_o} \approx -g\left(\frac{p^*}{p_o} - \frac{T_v^*}{T_{v,o}}\right) \qquad (2)$$

- More generally to take the hydrometeors into account:

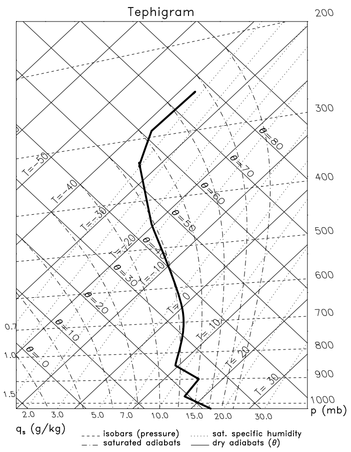
$$\rho = \rho_a(1+q_H) \qquad (3)$$

where q_H = mass mixing ratio of hydrometeors

- So that

$$B = g\left(\frac{T^*}{T_o} - \frac{p^*}{p_o} + 0.61q_v^* - q_H\right) \qquad (4)$$

Tephigram



A tephigram is useful for thunderstorm forecasts

- Exercise: In this diagram an environmental temperature profile is shown
- Use that to indicate the levels that are absolutely (dry) stable and unstable
- How is Θ changing with height in a stable/ unstable environment?

Buoyant instabilities

- The buoyancy restoring force produces stable sinusoidal oscillations in the **vertical** (undamped harmonic oscillation) when the potential temperature of a hydrostatically balanced mean state increases with height ($\partial\Theta/\partial z > 0$):

$$\frac{d^2w}{dt^2} + wN^2 = 0. \quad (5)$$

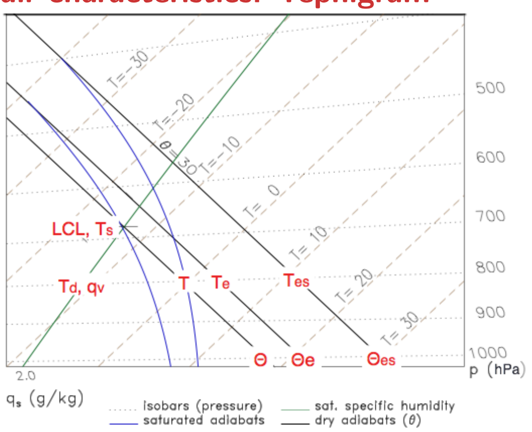
where $N \equiv \sqrt{\frac{g}{\Theta}\frac{\partial\Theta}{\partial z}}$ = Brunt-Väisälä frequency

- solution:

$$w = \hat{w} exp\left\{i\sqrt{N^2}t\right\} \quad (6)$$

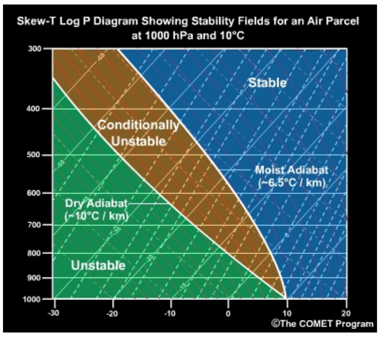
- $\tau \sim 8$ min in stable environment
- if $\partial\Theta/\partial z < 0$ ($N^2 < 0$), the solution may grow exponentially $\rightarrow$ buoyant instability

Moist air characteristics: Tephigram



LCL = lifting condensation level; Td = dew point; qv = specific humidity; Te=equivalent temp; Tes=saturated equivalent temp.; Θ,Θe,Θes = potential, equiv. and sat. equiv. pot. temp

Stability criteria for an air parcel in moist air

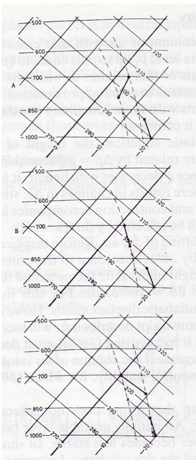


where $\Theta_{es} \equiv \Theta exp\{Lq_{vs}/c_p T\}$ = saturation equivalent potential temperature

(as opposed to $\Theta_e \equiv \Theta exp\{Lq_v/c_p T_s\}$)

Potential instability (lifting of layers)

(in terms of Θ_e)



lifting a whole layer of moist but unsaturated air ($\partial\Theta_e/\partial z < 0$) may be made **unstable** by lifting to saturation

also called convective instability (Figure: Rogers&Yau, 1989).

Inertial instabilities (= dynamic instability)

- Here use the absolute momentum $M = v + fx$ as a conserved property, i.e. $\frac{dM}{dt} = 0$
- analogous to Θ being conserved under adiabatic conditions and gravity and the vertical gradient of Θ determining the frequency of the buoyancy oscillation
- The Coriolis restoring force produces stable sinusoidal oscillations in the **horizontal** (undamped harmonic oscillation) when the absolute momentum of the 2D geostrophically balanced mean state increases with x ($\partial M/\partial x > 0$):

$$\frac{d^2u}{dt^2} + uf\frac{\partial M}{\partial x} = 0. \qquad (7)$$

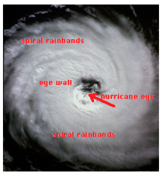
- solution:

$$u' = exp\{ivt\}, \ v = \pm\sqrt{f\partial M/\partial x} \qquad (8)$$

- $\partial M/\partial x > 0$: inertial oscillation; $\partial M/\partial x < 0$: inertial instability

Symmetric instability

- In the atmosphere buoyancy and the Coriolis force act simultaneously
- Assume that the large-scale mean flow is in geostrophic and hydrostatic balanced in the absence of friction
- The atmosphere may be stable for pure horizontal displacement and pure vertical displacement but **unstable** to slantwise displacement



Symmetric instability can be responsible for rainbands (*http : //ww2010.atmos.uiuc.edu/guides/mtr/hurr/gifs/def1.gif*)

Symmetric instability

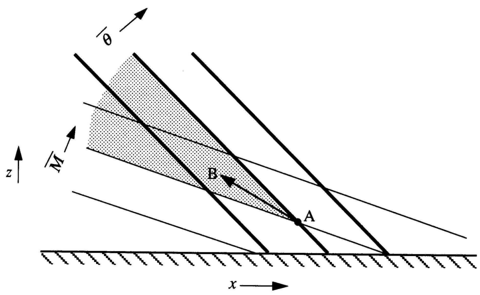


Figure 2.3 A symmetrically unstable two-dimensional mean state. The isolines of $\bar{\theta}$ and $\bar{M}$ in the x–z plane slope such that $\partial\bar{\theta}/\partial z > 0$ and $\partial\bar{M}/\partial x > 0$. A parcel lifted from A to B has a higher $\bar{\theta}$ and a lower $\bar{M}$ than its environment.

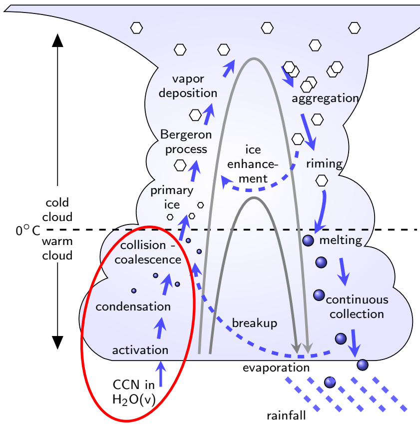
Symmetric instability

- conditions for symmetric instability: slope of the M surfaces must be less than the slope of the Θ surfaces:

$$\left.\frac{\partial \Theta}{\partial z}\right|_M < 0 \leftrightarrow \left.\frac{\partial M}{\partial x}\right|_\Theta < 0 \tag{9}$$

- Moist but unsaturated air maybe conditionally symmetric unstable (analogous to conditional instability) $\longrightarrow$ important for hurricane formation
- If lapse rate is conditionally unstable on a constant M surface, then replace Θ in above equation with Θ_{es}

Warm cloud processes



Phase changes

- Are easiest understood in terms thinking that a system wants to minimise its Gibbs free energy G (analogous to a system wanting to maximize its entropy)
-
$$G = u + e_s\alpha - Ts \tag{10}$$
where u = internal energy, e_s = saturation vapor pressure, α = specific volume, s = entropy
- nucleation from the vapor phase requires to form a new surface, which needs energy.
- if the vapor is supersaturated and the volume term of the change in Gibbs free energy is larger than the surface term, **nucleation** has occurred and the particles is said to be **activated**

Gibbs free energy for homogeneous nucleation

(Fig. 9.10 Seinfeld and Pandis, 1997)

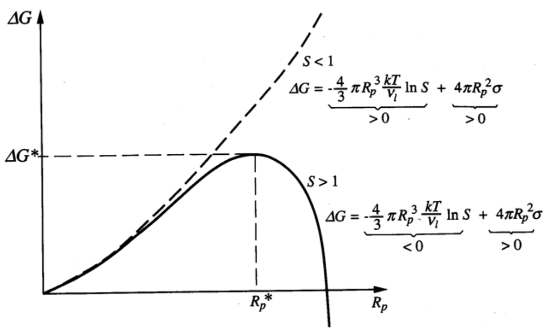


FIGURE 9.10 Gibbs free energy change for formation of a droplet of radius R_p from a vapor with saturation ratio S.

Kelvin equation

Vapor pressure enhancement over smaller drops due to surface tension

$$e_s(r) = e_s(\infty) exp\left(\frac{2\sigma}{R_v \rho_w T r}\right) \qquad (11)$$

where T = temperature, r = particle radius, σ = surface tension $\approx$ 0.075 N/m, ρ_w = water density, R_v = gas constant of water vapor (461.5 J kg^{-1} K^{-1}).

Saturation ratio S	Critical radius r*(μm)	number of molecules n
1	∞	∞
1.01	0.12	2.47 x 10^8
1.1	0.0126	2.81 x 10^5
2	1.73 x 10^{-3}	730
10	5.22 x 10^{-4}	20

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Raoult's Law

Vapor pressure reduction due to the presence of a non-volatile solute:

$$\frac{e'}{e_s(\infty)} = \frac{n_o}{i\,n + n_o} \sim 1 - \frac{i\,n}{n_o} = 1 - \frac{3imM_w}{4\pi M_s \rho_w r^3} \qquad (12)$$

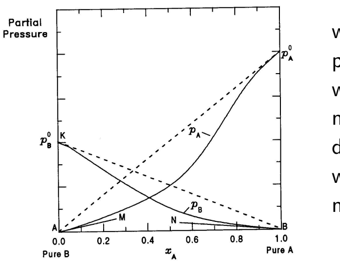

FIGURE 9.3 Equilibrium partial pressures of the components of a nonideal mixture of A and B. Dashed lines correspond to ideal behavior.

where e' = equilibrium vapor pressure over a solution with n_o water molecules and n solute molecules; i = degree of ionic dissociation; $M_{w,s}$ = molecular weight of water or solute; m = mass of solute.

(Seinfeld and Pandis, 1997)

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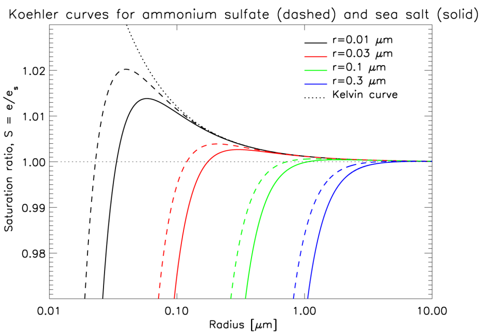
Köhler curve

Combination of Kelvin and Raoult's equation:

$$\frac{e'(r)}{e_s(\infty)} = 1 + \frac{2\sigma}{\rho_w R_v T r} - \frac{3imM_w}{4\pi M_s \rho_w r^3} \qquad (13)$$



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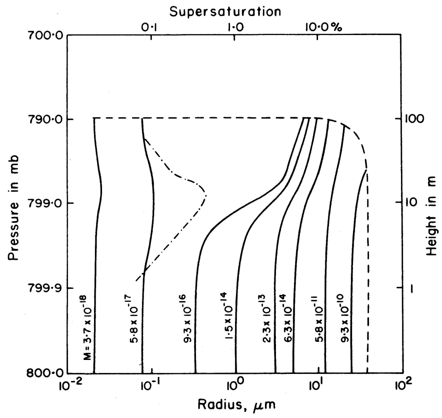


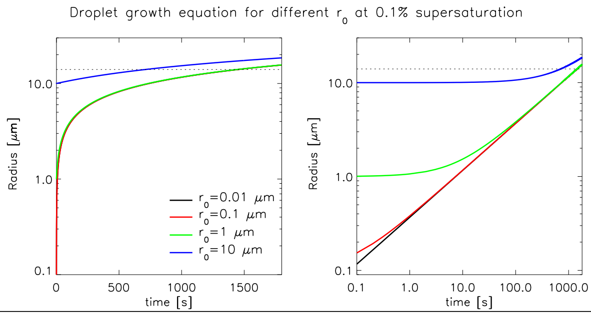
FIG. 7.3. Initial formation of cloud droplets and the variation of super-saturation above cloud base. (Adapted from Mordy, 1959.)

Figure: Rogers and Yau, 1989

Droplet growth equation

$$r\frac{dr}{dt} = \frac{S-1}{\left(\frac{L}{R_v T} - 1\right)\frac{L\rho_w}{KT} + \frac{\rho_w R_v T}{De_s}} = \frac{S-1}{F_k + F_d} \qquad (14)$$

where F_k = thermodynamic term, F_d = vapor diffusion term, K = thermal conductivity of air, D = diffusion coefficient of water vapor in air



Growth by collection (=collision-coalescence)

- coalescence = one or two particles merge during contact

$$\frac{dR}{dt} = \frac{\pi}{3}\int_o^R \left(\frac{R+r}{R}\right)^2 n(r)[u(R) - u(r)]r^3 E(R,r)dr \qquad (15)$$

where R, r = radius of collector and collected drop; $n(r)$ = number of drops with size r

- $u(R), u(r)$ = fall velocity of collector and collected drop
- $E(R,r)$ = collision efficiency (= fraction of drops with radius r in the path swept out by collector drop that actually collide with it):

$$E(R,r) = \frac{x^2}{(R+r)^2} \qquad (16)$$

Collision efficiency

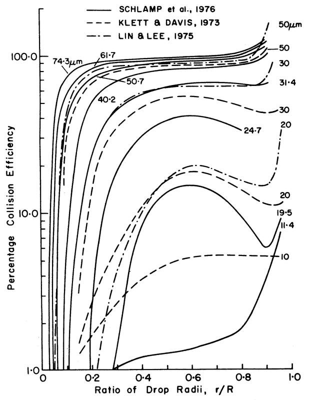


FIG. 8.2. Computed collision efficiencies for pairs of drops as a function of the ratio of their radii. Curves are labeled according to the radius *R* of the larger drop.

Figure: Rogers and Yau, 1989

Cloud drop/rain fall speed

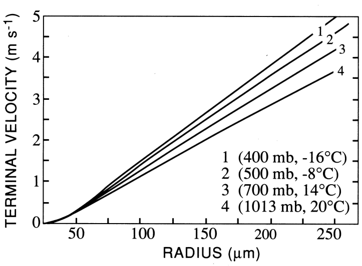


Figure 3.2 Fall velocity of water drops <500 μm in radius for various atmospheric conditions. (From Beard and Pruppacher, 1969. Reprinted with permission from the American Meteorological Society.)

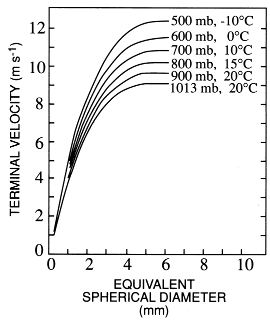


Figure 3.3 Fall velocity of water drops >500 μm in radius. (From Beard, 1976. Reprinted with permission from the American Meteorological Society.)

Figure: Houze, 1993

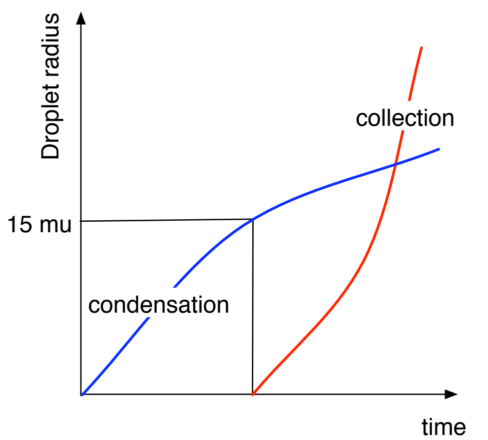


Figure: Droplet growth by condensation versus collection