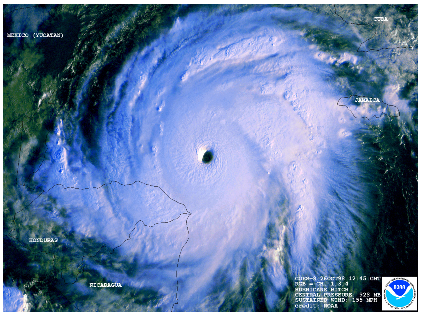
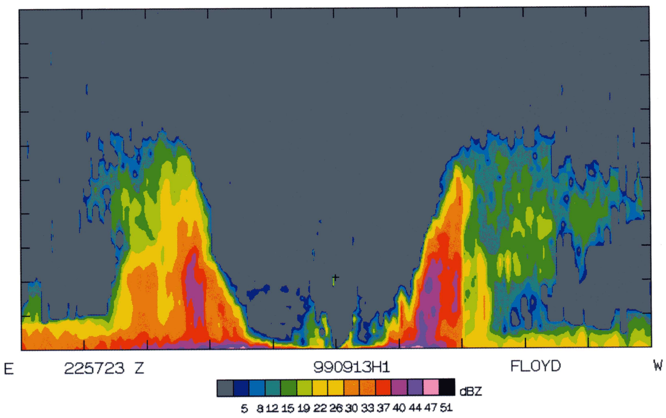
Hurricanes II: Steady state model



http://www.osei.noaa.gov/Events/Tropical/Atlantic/1998/Mitch_10/TRCmitch299B_G8.jpg

Motivation



Emanuel, 2005

Summary of first part

- Primary circulation of TC's can be described by gradient wind balance
- A warm core of TC's can be found, this is consistent with the primary circulation
- TC's can be described using the Carnot process:
  - Heating due to moisture from the boundary layer
  - Maximal tangential wind speed can be estimated from the thermodynamic properties
  - Pressure deviation in the core of TC's can be estimated from the maximal tangential wind speed

Today's lecture: Steady state model (Emanuel, 1986) in details (tangential wind, temperature and momentum distributions)

Next lectures:

- Change of intensity/tracks of hurricanes
- Models and forecasts

Equations

Momentum equations, continuity equation and thermodynamic equation in cylindrial polar coordinates (r, ϑ, z) on an f–plane:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \vartheta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \vartheta} + w\frac{\partial v}{\partial z} - \frac{uv}{r} + fu = -\frac{1}{r\rho}\frac{\partial p}{\partial \vartheta} \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \vartheta} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \tag{3}$$

$$\frac{1}{r}\frac{\partial \rho r u}{\partial r} + \frac{1}{r}\frac{\partial \rho v}{\partial \vartheta} + \frac{\partial \rho r w}{\partial z} = 0 \tag{4}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial r} + \frac{v}{r}\frac{\partial \theta}{\partial \vartheta} + w\frac{\partial \theta}{\partial z} = \dot{\theta} \tag{5}$$

Equations

Exner–function

$$\pi \equiv \left(\frac{p}{p_o}\right)^{R/c_p} , \quad T = \pi\theta \tag{6}$$

Hydrostatic approximation:

$$\frac{\partial p}{\partial z} = -\rho g \overset{ideal}{\underset{gas}{\Leftrightarrow}} \frac{d\ \log \pi}{dz} = \frac{g}{c_p T} \tag{7}$$

Absolute angular momentum m:

$$m \equiv rv + \frac{1}{2}fr^2 \tag{8}$$

Equation for evolution of m

$$\frac{\partial m}{\partial t} + u\frac{\partial m}{\partial r} + \frac{v}{r}\frac{\partial m}{\partial \vartheta} + w\frac{\partial m}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial \vartheta} \tag{9}$$

For an axisymmetric flow without friction: $-\frac{1}{\rho}\frac{\partial p}{\partial \vartheta} = 0$

Notations

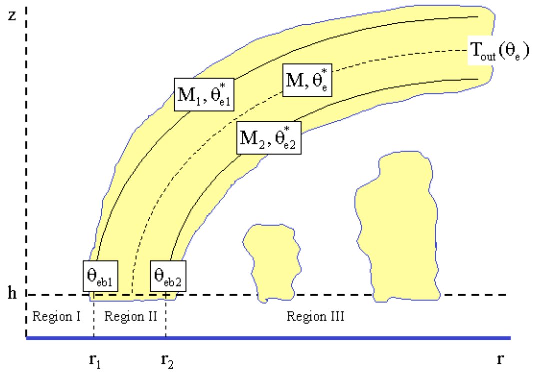
In this and the following lectures we use this terminology:

- $*$ denotes saturated quatities (e.g. saturated equivalent potential temperature θ_e^*, saturation specific humidity q^*)
- subscript $_s$ denotes quantities at the surface, i.e. at $z = 0$

Steady state model

Question: Why does the eye (radius) increase with height?

For answering this question we regard the steady state model of a TC:



after Emanuel, 1986

Steady state model - basic assumptions

Basic assumption: Steady state TC with

- axisymmetric circulation: $\frac{\partial}{\partial \theta} = 0$
- hydrostatic balance: $\frac{\partial p}{\partial z} = -g\rho$
- gradient wind balance: $\frac{v^2}{r} + fv = \frac{1}{\rho}\frac{\partial p}{\partial r}$
- vortex neutral to slantwise convection (symmetric stability), i.e. combined buoyant/centrifugal potential of boundary layer air is zero:

$$\left.\frac{\partial \theta^*_e}{\partial z}\right|_m = 0, \quad \left.\frac{\partial m}{\partial r}\right|_{\theta^*_e} = 0 \tag{10}$$

where $m = rv + \frac{1}{2}fr^2$ denotes the total angular momentum.

Recap of symmetric instability

Two kinds of stabilities:

- Buoyant stability: $\frac{\partial \theta}{\partial z} > 0$, restoring force = gravity
- Inertial stability: $\frac{\partial M}{\partial x} > 0$, restoring force = Coriolis force

In the atmosphere, buoyancy and Coriolis act simultaneously. The atmosphere may be stable for pure vertical and horizontal displacements, but **unstable** to slantwise displacement

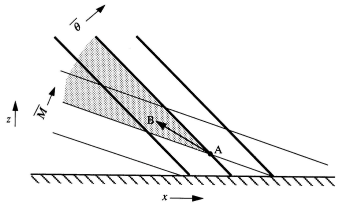


Figure 2.3 A symmetrically unstable two-dimensional mean state. The isolines of $\bar{\theta}$ and $\bar{M}$ in the x–z plane slope such that $\partial\bar{\theta}/\partial z > 0$ and $\partial\bar{M}/\partial x > 0$. A parcel lifted from A to B has a higher $\bar{\theta}$ and a lower $\bar{M}$ than its environment.

Houze, 1993

Thread for the lecture

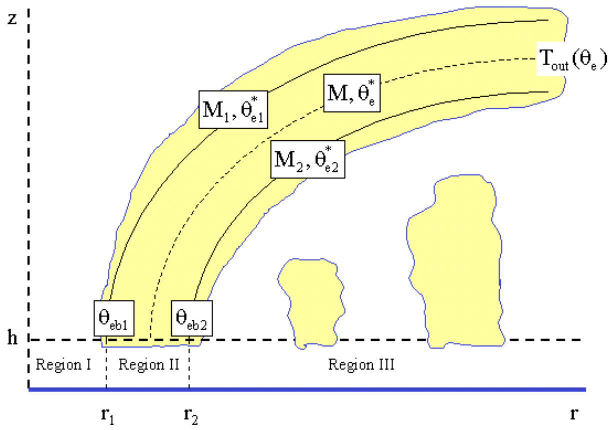
Describing the properties of the TC in different regions:

- Above the boundary layer, i.e. slope of the eyewall
- Boundary layer
  - I eye region (centre pressure)
  - II eye wall
  - III outer region (inflow region)

General question:

How are dynamics and thermodynamics related in a TC?

Thread for the lecture



Angular momentum

The angular momentum $m = rv + \frac{1}{2}fr^2$ is conserved in an axisymmetric framework (see last lecture, slide 15, eq. (9)):

$$\frac{\partial m}{\partial t} + u\frac{\partial m}{\partial r} + \frac{v}{r}\frac{\partial m}{\partial \vartheta} + w\frac{\partial m}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial \vartheta} = 0 \text{ for an axisymmetric model}$$

A small perturbation dm (on an isosurface of m) splitted in r and p direction yields:

$$dm = \frac{\partial m}{\partial r}dr + \frac{\partial m}{\partial p}dp = 0 \Leftrightarrow \left.\frac{dr}{dp}\right|_m = -\frac{\frac{\partial m}{\partial p}}{\frac{\partial m}{\partial r}} \tag{11}$$

and this is the slope of an m–surface.
Question: How can the m-surfaces be connected to other variables of the TC?

Some dynamics

We start with the hydrostatic and gradient wind balance ($\alpha = 1/\rho$):

$$\alpha \frac{\partial p}{\partial z} = -g \tag{12}$$

$$\alpha \frac{\partial p}{\partial r} = \frac{v^2}{r} + fv = \frac{m^2}{r^3} - \frac{1}{4} f^2 r \tag{13}$$

These equations can be reformulated to

$$g \left.\frac{\partial z}{\partial p}\right|_r = -\alpha \tag{14}$$

$$g \left.\frac{\partial z}{\partial r}\right|_p = \frac{m^2}{r^3} - \frac{1}{4} f^2 r \tag{15}$$

and by applying $\frac{\partial}{\partial r}$ to eq.(14) and $\frac{\partial}{\partial p}$ to eq.(15) this yields the thermal wind equation

$$\frac{1}{r^3} \left.\frac{\partial m^2}{\partial p}\right|_r = -\left.\frac{\partial \alpha}{\partial r}\right|_p \tag{16}$$

Some thermodynamics

We assume reversible thermodynamics, i.e. $\alpha = \alpha(p, s^*)$ with s^* moist saturated entropy ($s^* := c_p \log \theta_e^*$):

$$\left.\frac{\partial \alpha}{\partial r}\right|_p = \left.\frac{\partial \alpha}{\partial s^*}\right|_p \left.\frac{\partial s^*}{\partial r}\right|_p \tag{17}$$

Using the (saturated) moist static energy (or enthalpy) $h = c_v T + p\alpha + Lq_{vs}$, $dh = Tds^* + \alpha dp$ we see:

$$\left.\frac{\partial h}{\partial p}\right|_{s^*} = \alpha, \left.\frac{\partial h}{\partial s^*}\right|_p = T \tag{18}$$

and therefore

$$\left.\frac{\partial \alpha}{\partial s^*}\right|_p = \frac{\partial^2 h}{\partial p \partial s^*} = \frac{\partial^2 h}{\partial s^* \partial p} = \left.\frac{\partial T}{\partial p}\right|_{s^*} \quad \text{moist adiabatic T gradient} \tag{19}$$

From the definition of s^* we find:
lines of constant θ_e^* are lines of constant s^*

Interpretation

Then eq. (16) reads as

$$\frac{2m}{r^3} \left.\frac{\partial m}{\partial p}\right|_r = \frac{1}{r^3} \left.\frac{\partial m^2}{\partial p}\right|_r = -\left.\frac{\partial T}{\partial p}\right|_{s^*} \left.\frac{\partial s^*}{\partial r}\right|_p \tag{20}$$

We assume neutrality for slantwise convection, this implies that air from the PBL (which is neutrally buoyant) lifted along surfaces of constant angular momentum remains neutrally buoyant
$\Leftrightarrow$ moist entropy of lifted parcels equals the saturated entropy of the environment.
This implies that the saturated moist entropy s^* does NOT vary along angular momentum surfaces m and s^* is a function of m

$$s^* = s^*(m) \tag{21}$$

Interpretation

This yields a reformulation of eq.(16):

$$\frac{2m}{r^3}\frac{\partial m}{\partial p}\Big|_r = \frac{1}{r^3}\frac{\partial m^2}{\partial p}\Big|_r = -\frac{\partial T}{\partial p}\Big|_{s^*}\frac{ds^*}{dm}\frac{\partial m}{\partial r}\Big|_p \tag{22}$$

here we used, that s^* depends only on m, i.e.: $\frac{ds^*}{dm} = \frac{\partial s^*}{\partial m}$
For determing the slope of the eyewall, we note, that along an m-surface:

$$0 = dm = \frac{\partial m}{\partial r}dr + \frac{\partial m}{\partial p}dp \Rightarrow \frac{dr}{dp} = -\frac{\frac{\partial m}{\partial p}}{\frac{\partial m}{\partial r}} \tag{23}$$

This leads to an equation for the slope of the m–surfaces (in $r-p$–space):

$$\frac{dr}{dp}\Big|_m = \frac{r^3}{2m}\frac{ds^*}{dm}\frac{\partial T}{\partial p}\Big|_{s^*} \tag{24}$$

Slope of the eyewall

Slope of the m–surfaces in the $r-p$–space

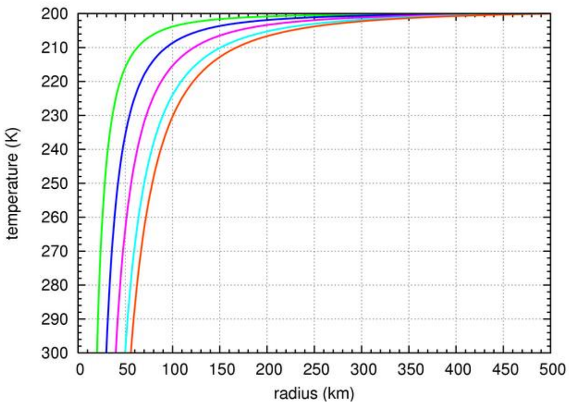
$$\frac{dr}{dp}\Big|_m = \frac{r^3}{2m}\frac{ds^*}{dm}\frac{\partial T}{\partial p}\Big|_{s^*} \tag{25}$$

By integrating this equation along an m–surface
($r \to r_o, T \to T_o, p \to p_o$) we get:

$$\frac{1}{r^2}\Big|_m - \frac{1}{r_o^2}\Big|_m = -\frac{1}{m}\frac{ds^*}{dm}(T - T_o(s^*, p_o)) \tag{26}$$

where the integration constant T_o may be interpreted as "outflow temperature" This equation yields the shape (in $r-T$–space) of m (or s^*) – surfaces

Slope of the eyewall



Relation $r-T$ on constant angular momentum surfaces for $ds^*/dm = const$

Boundary layer

The equation above reads along the top of the PBL ($z = h$, assuming $r << r_o$):

$$-r^2\Big|_m \frac{ds^*}{dm}(T_B - T_o(s^*, p_o)) = m \text{ at } z = h \tag{27}$$

where T_B denotes the temperature inside the boundary layer (assumption: T_B constant for $0 \le z \le h$, in good agreement with observations)
Multiplying this equation by $\frac{\partial m}{\partial r}$ results in

$$-r^2\frac{\partial s^*}{\partial r}(T_B - T_o) = \frac{1}{2}\frac{\partial m^2}{\partial r} \text{ at } z = h \tag{28}$$

Goal: From this equation we want to derive a relationship between θ_e and p, i.e. we must eliminate m

Boundary layer

Using the Exner-function $\pi = \left(\frac{p}{\hat{p}}\right)^{R/c_p}$ the gradient wind balance can be written as follows:

$$m^2 = r^3\left(c_p T_B \frac{\partial \log \pi}{\partial r} + \frac{1}{4}f^2 r\right) \tag{29}$$

inserting this into eq.(28) yields at $z = h$:

$$-\frac{T_B - T_o}{T_B}\frac{\partial \log \theta_e}{\partial r} = \frac{\partial \log \pi}{\partial r} + \frac{1}{2}\frac{\partial}{\partial r}\left[r\frac{\partial \log \pi}{\partial r}\right] + \frac{1}{4}\frac{rf^2}{c_p T_B} \tag{30}$$

This equation can be integrated from the radial extent of the storm r_o to r:

$$-\frac{T_B - \overline{T}_o}{T_B}\log\left(\frac{\theta_e}{\theta_{eo}}\right) = \log\left(\frac{\pi_{eo}}{\pi_e}\right) + \frac{1}{2}\left[r\frac{\partial \log \pi}{\partial r}\right]\Big|_{r_o} - \frac{1}{2}\left[r\frac{\partial \log \pi}{\partial r}\right] + \frac{1}{4}\frac{f^2}{c_p T_B}(r^2 - r_o^2) \tag{31}$$

Boundary layer

Here,

$$\overline{T}_o := \frac{1}{\log(\theta_e^*/\theta_{ea})}\int_{\log \theta_{ea}}^{\log \theta_e} T_o d \log \theta_e^* \tag{32}$$

is the average outflow temperature weighted with the saturated moist entropy of the outflow angular momentum surfaces.
By rewriting eq. (31) and integrating the eq. from $r = 0$ to the radial extent of the storm $r = r_o$, this yields a relationship between the geometric area of the storm and its areal-average boundary layer moist entropy surfeit:

$$r_o^2 = \frac{16 c_p T_B}{f^2}\frac{1}{r_o^2}\int_0^{r_o}\frac{T_B - \overline{T}_o}{T_B}\log\frac{\theta_e}{\theta_{ea}} r dr \tag{33}$$

Estimation of the central pressure

At the storm centre π_c ($r_c = 0$) equation (31) implies:

$$\log\left(\frac{\pi_c}{\pi_a}\right) = -\frac{T_B - \overline{T}_o}{T_B}\log\left(\frac{\theta_{ec}}{\theta_{ea}}\right) + \frac{1}{4}\frac{f^2}{c_p T_B} r_o^2 \qquad (34)$$

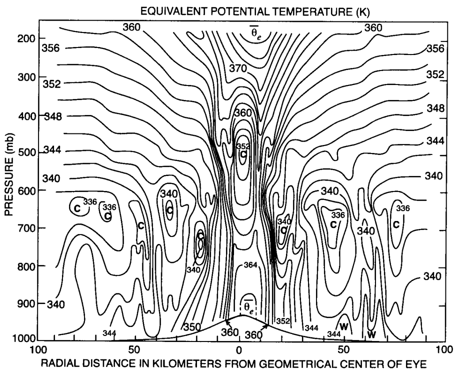
Interpretation: Pressure deficit may be expected to be weaker in geometrically larger storms (noticable for $r_o \leq 500$ km).
estimation: $\frac{\theta_{ec}}{\theta_{ea}} \approx 1$, $\frac{\pi_c}{\pi_a} \approx 1$, therefore almost linear relationship between pressure deficit and θ_e^*:

$$p'_c \approx -\frac{c_p}{R} p_a \frac{T_B - \overline{T}_o}{T_B}\frac{\theta'_{ec}}{\theta_{ea}} \qquad (35)$$

For $\theta_{ea} = 345$ K, $p_o = 1015$ hPa, $T_B = 295$ K and $T_o = 200$ K this yields

$$p'_c[hPa] \approx -3.3\theta'_{ec}[K] \qquad (36)$$

Potential temperature and pressure



Hawkins and Imbembo, 1976

Estimation: $\theta_{ec} \approx 365 - 345$ K, hence $p'_c \approx -66$ hPa

Boundary layer

The boundary layer can be divided into three regions:

I eye region
II eyewall
III outer region

We use a streamfunction ψ to decribe the velocities:

$$\rho r u = -\frac{\partial \psi}{\partial z}, \quad \rho r v = \frac{\partial \psi}{\partial r} \qquad (37)$$

Let c be a quantity that is conserved (e.g. $c = \theta_e, m, \ldots$), then the following equation holds:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial r} + w\frac{\partial c}{\partial z} = -\frac{1}{\rho}\frac{\partial \tau_c}{\partial z} \qquad (38)$$

where τ denotes the vertical flux of c for all processes **excluding** the mean circulation

Boundary layer

Assumptions:

- storm in steady state: $\frac{\partial c}{\partial t} = 0$
- PBL well mixed: $\frac{\partial c}{\partial z} = 0$

$$u\frac{\partial c}{\partial r} = -\frac{1}{\rho}\frac{\partial \tau_c}{\partial z} \Leftrightarrow -\frac{1}{r\rho}\frac{\partial \psi}{\partial z}\frac{\partial c}{\partial r} = -\frac{1}{\rho}\frac{\partial \tau_c}{\partial z} \Leftrightarrow \frac{\partial \psi}{\partial z}\frac{\partial c}{\partial r} = r\frac{\partial \tau_c}{\partial z} \quad (39)$$

Integration along z yields

$$\int_0^h \frac{\partial \psi}{\partial z}\frac{\partial c}{\partial r}dz = \int_0^h r\frac{\partial \tau_c}{\partial z}dz \Leftrightarrow \psi\frac{\partial c}{\partial r}\Big|_0^h = r\tau_c\Big|_0^h \quad (40)$$

$$\Leftrightarrow \psi(h)\frac{\partial c}{\partial r}\Big|_h - \underbrace{\psi(0)}_{=0}\frac{\partial c}{\partial r}\Big|_0 = r(\tau_c(h) - \tau_c(0)) \quad (41)$$

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Boundary layer - Region II

We assume $\tau_c(h) \approx 0$, i.e. a negligible flux at the top of the PBL. From standard aerodynamics the following equation holds for the surface flux over the ocean:

$$\tau_c(0) = -\rho C_c|\vec{v}|(c(h) - c(0)) \quad (42)$$

and this reads for $c = s^*$ and $c = m$ ($\approx rv$, neglecting Coriolis term):

$$\begin{aligned}\tau_{s^*} &= -\rho C_s|\vec{v}|(s^*(h) - s^*(0)) = -\rho C_s|\vec{v}|c_p(\log\theta_e^*(h) - \theta_e^*(0))\\ \tau_m &= -\rho C_m|\vec{v}|(m(h) - m(0)) = -\rho C_m|\vec{v}|r\vec{v}(h)\end{aligned}$$

From these equations we can derive an expression for $\frac{ds^*}{dm}$

$$\frac{ds^*}{dm} = \frac{\partial s^*}{\partial m}\Big|_h = \frac{\frac{\partial s^*}{\partial r}\Big|_h}{\frac{\partial m}{\partial r}\Big|_h} = \frac{\tau_{s^*}}{\tau_m}\Big|_{z=0} \quad (43)$$

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Boundary layer - Region II

Remember eq.(27):

$$-r^2\Big|_m \frac{ds^*}{dm}(T_B - T_o(s^*, p_o)) = m \text{ at } z = h$$

Inserting $\frac{ds^*}{dm} = \frac{\tau_{s^*}}{\tau_m}\Big|_{z=0}$ in this eq. and using the definition of m we derive:

$$\log\theta_e = \log\theta_{es}^* - \frac{C_m}{C_s}\frac{1}{c_p(T_B - T_o)}\left(v^2 + \frac{1}{2}frv\right) \quad (44)$$

From this relation we can derive the following estimation for the wind speed (under the assumption $rf << v$)

$$v^2 \approx \frac{C_s}{C_m}c_p(T_B - T_o)\log\left(\frac{\theta_e}{\theta_{es}^*}\right) \quad (45)$$

and this is consistent with the estimation of the maximal wind speed from the energetics of a Carnot process (see last lecture).

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Carnot process

$$|\vec{V}_m|^2 \approx \frac{C_k}{C_D}\epsilon T_B(s_S^* - s_B)\Big|_m; \ \epsilon = \frac{T_B - T_{out}}{T_B}, \ s = c_p \log\theta_e \quad (46)$$

Remarks:

- The efficiency of a Carnot–process can be measured by the quantity $\epsilon = \frac{T_B - T_{out}}{T_B}$
- For $T_B = 295$ K, $T_o = 200$ K this yields an efficiency $\epsilon \approx 0.32$
- Compare this to the efficiency of a car motor ($\eta \approx 0.56$) or a fridge ($\eta \approx 0.1$)

Boundary layer - the role of humidity

Assumption: θ_e is vertically uniform in PBL, i.e. $\theta_e = \theta_{es}$ (surface) using the definition of $\theta_e = \frac{T}{\pi}\exp\left(\frac{Lq}{c_p T}\right)$ we can derive:

$$\begin{aligned}
\log\frac{\theta_e}{\theta_{ea}}\Big|_{z=h} &= -\log\frac{\pi_s}{\pi_{sa}} + \left(\frac{L}{c_p T_S}(q - q_a)\right)\Big|_s && (47)\\
&= -\log\frac{\pi_s}{\pi_{sa}} + \left(\frac{L}{c_p T_S}(q^* RH - q_a^* RH_a)\right)\Big|_s && (48)
\end{aligned}$$

with constant sea surface temperature T_S and the ambients states (subscript "a"); q^* denotes the saturation humidity. From the hydrostatic balance we can derive

$$\frac{\partial \log\pi}{\partial z} = \frac{g}{c_p T} \Leftrightarrow \log\frac{\pi}{\pi_s} = \int_{z=0}^{z=h}\frac{g}{c_p T}dz = \log\frac{\pi_a}{\pi_{sa}} \quad (49)$$

Boundary layer - the role of humidity

The saturation specific humidity q^* can be approximated as follows:

$$\begin{aligned}
q^* = q_a^*\frac{p_a}{p} = q_a^*\left(\frac{\pi}{\pi_a}\right)^{-\frac{c_p}{R}} &= q_a^*\exp\left(-\frac{c_p}{R}\log\frac{\pi}{\pi_a}\right) && (50)\\
&\approx q_a^*\left(1 - \frac{c_p}{R}\log\frac{\pi}{\pi_a}\right) && (51)
\end{aligned}$$

This yields the following relation for the boundary layer:

$$\log\frac{\theta_e}{\theta_{ea}}\Big|_{z=h} = -\log\frac{\pi_s}{\pi_{sa}}\left(1 + \frac{Lq_a^* RH}{RT_S}\right) + \frac{Lq_a^*}{c_p T_S}(RH - RH_a) \quad (52)$$

Region I (eye)

- Assumption of negligible flux of m, s^* at top of the PBL does not hold, but
- inside the eye the m and s^* surfaces coincide due to the "quasi" solid body rotation, i.e. eq.(44) also holds
- From eq.(52) we can derive that the relative humidity inside the eye increases inwards
- Combining eq.(52) with the relation for the central pressure eq.(34) this yields an explicit relation between central pressure and relative humidity

$$\log \frac{\pi_{sc}}{\pi_{sa}} \approx \frac{-\epsilon \frac{L q_a^*}{c_p T_S}(RH_c - RH_a) + \frac{1}{4}\frac{f^2 r_o^2}{c_p T_B}}{1 - \epsilon\left(1 + \frac{L q_a^* RH_c}{R T_S}\right)} \tag{53}$$

where $\epsilon = (T_B - \overline{T}_o)/T_B$

Region I - Upper bound of pressure deficit

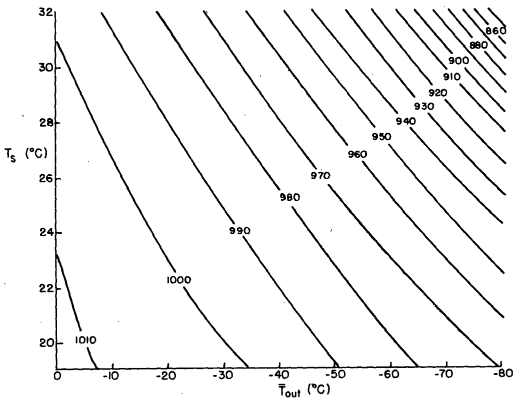


FIG. 2. Minimum attainable central surface pressure (mb) computed from (26) as a function of surface air temperature (T_s) and weighted mean outflow temperature ($\overline{T}_{out}$) assuming an ambient surface pressure of 1015 mb, ambient surface relative humidity of 80%, f evaluated at 20 degrees latitude, and $r_0 = 500$ km.

Emanuel, 1986

Region III (outer region)

The assumption of a negligible flux (of quantities s^*, m) at the top of the PBL does not hold for the outer region, hence the equations derived for the eyewall region do not hold
However, we can assume that due to the exchange by the turbulent fluxes at the top of the PBL and the boundary layer induced subsidence the relative humidity remains nearly constant ($RH \equiv RH_a \approx 80\%$)

Region III (outer region)

From eq.(52) in combination of eq.(31) a relation between the radius of maximal wind speed and the outer radius can be derived:

$$r_o^{2+2\beta} \approx r_m^{2\beta} 2 \frac{T_B - T_o}{T_B} \frac{C_s}{C_m} \frac{T_B}{T_S} \frac{Lq_a^*}{f^2} (1 - RH_{as})(1+\beta) \qquad (54)$$

In the outer region the tangential wind speed is of the form

$$V \propto r^{-\beta}, \beta \equiv 1 - \frac{T_B - T_o}{T_B} \left(1 + \frac{Lq_a^* RH_{as}}{RT_S}\right) \approx 0.5 \qquad (55)$$

where r_o denotes outer radius, r_m radius of maximal wind speed, T_S surface temperature, T_B boundary layer temperature, T_o outflow temperature, $\epsilon = (T_B - T_o)/T_B$

Remark: The decay of v in the outer region is slower that with the Rankine model $v \propto r^{-1}$.

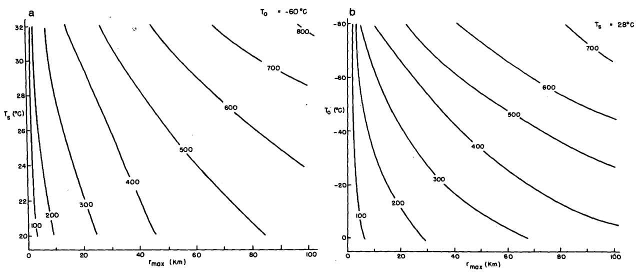
Region III (outer region)

From eq.(54) the relationship between the radii, latitude (f), surface temperature T_S, boundary layer temperature T_B and outflow temperature T_o (used in $\epsilon = (T_B - T_o)/T_B$) can be studied:

- As latitude increases (i.e. f) r_o becomes smaller and/or r_m becomes larger
- As T_S increases r_o increases and/or r_m decreases
- As T_o decreases r_o increases and/or r_m decreases

See figures for this relationship

Region III



Outer radius $r_o(km)$ as a function of maximum winds (i.e. r_{max}), surface temperature T_S and outflow temperature T_o Emanuel, 1986

Region III

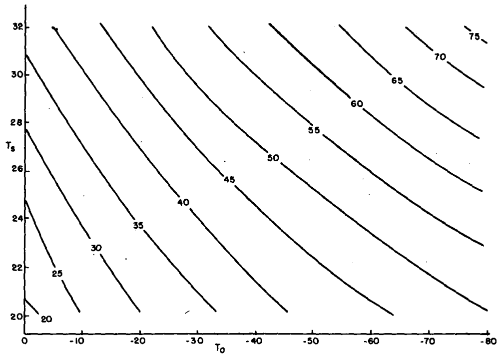
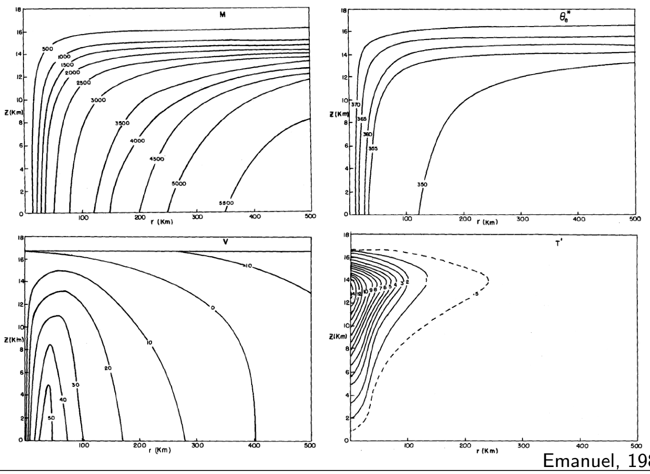


FIG. 6. Maximum gradient wind (m s^{-1}) as a function of surface air temperature (T_s, °C) and outflow temperature (T_0, °C) computed from (43) for the same conditions as used to construct Fig. 2 and $C_\theta = C_D$.

Emanuel, 1986

Complete solution – distributions



Emanuel, 1986

Complete solution – remarks

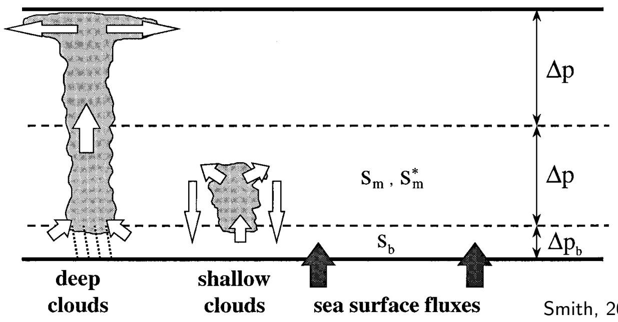
- Eyewall region is dominated by cyclone scale fluxes
- Outer region (outside the radius of maximal wind speed) is controlled by turbulent fluxes from/to the PBL.

Including cumulus parameterization

- Convection in the eye wall
- diabatic processes represented by saturated moist entropy (reversible)

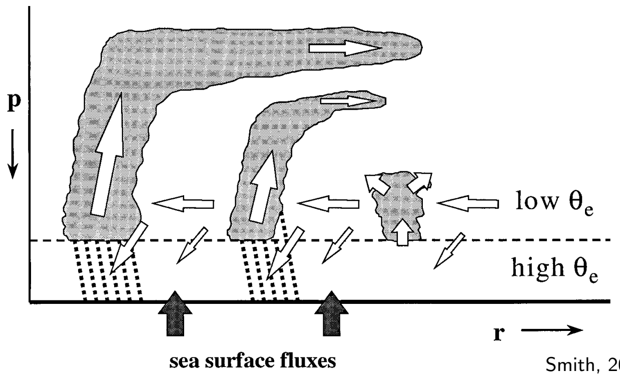
Cumulus convection redistribute heat acquired from sea surface to keep the environment locally neutral to slantwise convection ("quasi-equilibrium")
Closure problem = radial distribution of subcloud layer entropy
Extension of hurricane model with cumulus parameterisation (Emanuel, 1989; 1995)

Including cumulus parameterization



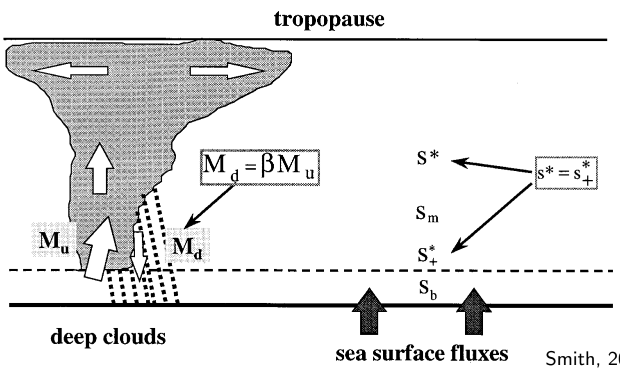
Smith, 2000

Including cumulus parameterization



Smith, 2000

Including cumulus parameterization



Smith, 2000