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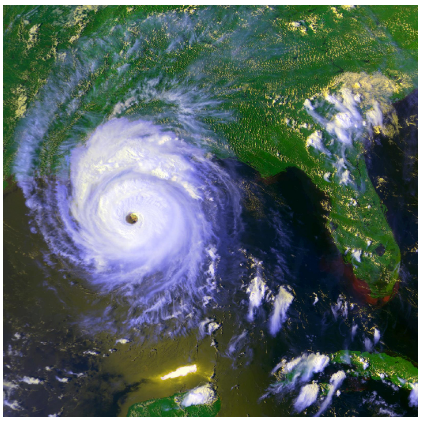
Hurricanes I



http://www.nwrc.usgs.gov/images/andrew.jpg

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Motivation

Darwin after Cyclone Tracy, 25/12/1974

http://www.austehc.unimelb.edu.au/fam/images/BXHM0344.jpg

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Definition

- A tropical cyclone is a storm system with a closed circulation around a center of low pressure, driven by heat energy released as moist air drawn in over warm ocean waters rises and condenses.
- The name underscores their origin in the tropics and their cyclonic nature. They are distinguished from extratropical storms by the heat mechanism that fuels them.
- Tropical cyclones encompass tropical depressions, tropical storms, hurricanes, and typhoons.

Name "hurricane" probably after the evil god of winds "Huracan" or "Jurakan" in the Mayan civilization

Huracan

left: likeness of the god Huracan (Cuban ceramic vase),
right: Universal symbol of the TC

Emanuel, 2005

Structure

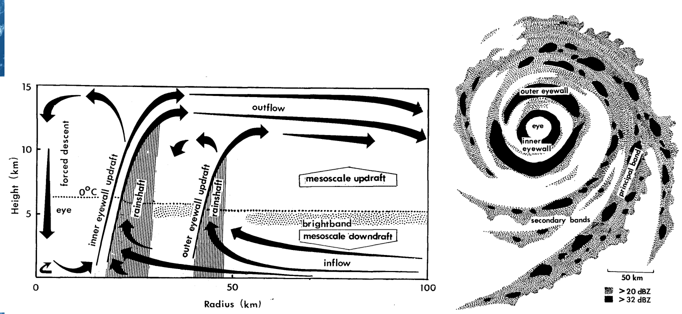
Mature Tropical Cyclone (TC):

- horizontal quasi–symmetric (primary) circulation
- superimposed thermally-direct vertical–radial (secondary) circulation

Schematic overview:

- “eye”
- eyewalls
- spiral bands → rain bands

Precipitation patterns



Willoughby, 1988

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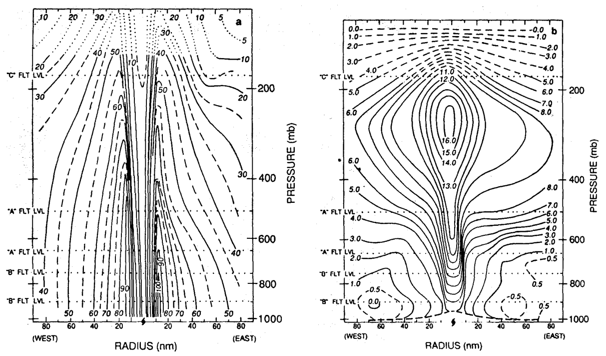
Wind and temperature distribution



Vertical cross–sections of azimutal wind and temperature,

Hawkins and Rubsam, 1968

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Strength, intensity and size

Definitions:

- intensity: cyclone core → maximum wind speed, minimum pressure
- strength: outer circulation → spatial–averaged wind speed over an anulus $100 < r < 250$ km
- size: averaged radius of gale force winds (≥ 17 m s^{-1}) or of the outer closed isobar

Remark: observations show that

- size and strength are correlated
- neither size nor strength is correlated to intensity

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Asymmetries

- inner–core region of intense TC is nearly axisymmetric
- the core is surrounded by a less symmetric outer vortex that merges into the synoptic environment

The asymmetries have significantly impact on storm motion
For TCs originating in the monsoon trough the asymmetric flow is often accompagnied with a band of convection that joins the TC with the trough

Formation

Distinction:

- core formation: existence of a (warm) core region that can be identified and classified as TC (i.e. mean maximal wind speed $> 17.5 \mathrm{m\ s}^{-1}$)
- Intensification: further development of the maximum windspeed

In the following: TC formation = transition from cloud cluster stage to TC stage with mean maximal wind speed exceeding $17.5 \mathrm{m\ s}^{-1}$
Changes in wind speed of the outer vortex are referred to as outer structure change/strength change/size change

Necessary conditions for TC formation

after Grey (1995) six factors for TC formation:

1. large values of low–level relative vorticity
2. location at least few degrees polewards of the equator (significant contribution of Coriolis force)
3. weak vertical shear of horizontal winds
4. Sea surface temperature (SST) larger than 26°C, deep thermocline ($>$ 50 m)
5. conditional instability through a deep atmospheric layer
6. large values of relative humidity in the lower and mid troposphere

factors 1-3: (horizontal) dynamics
factors 4-6: thermodynamics

Large scale conditions for TC formation

1. TCs form from preexisting disturbances containing abundant deep convection
2. Preexisting disturbances must aquire a warm core thermal structure throughout the troposphere
3. Formation is preceed by an increase of lower troposphere relative vorticity over a horizontal scale of $\sim 1000 - 2000$ km
4. Large scale environment with low wind shear
5. Early indicator for formation: Appearence of curved banding features of the deep convection in the incipient disturbance
6. Inner core may originate as a midlevel mesovortex that has formed in association with a preexisting area of altostratus (e.g. in the stratiform precipitation region of a MCS)

Questions to be answered:

- Wind circulations and pressure in a TC?
- Why is the pressure minimum in the center?
- Why has a TC a warm core (i.e. an eye)?
- Why does the eye increase with height?
- What is the dynamics of the eye?
- Why are rainbands asymmetric?
- How does a TC move (and why)?
- How are TC's simulated and forecasted?

Equations

Momentum equations, continuity equation and thermodynamic equation in cylindrial polar coordinates (r, ϑ, z) on an f–plane:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \vartheta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial r} \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \vartheta} + w\frac{\partial v}{\partial z} - \frac{uv}{r} + fu = -\frac{1}{r\rho}\frac{\partial p}{\partial \vartheta} \quad (2)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \vartheta} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad (3)$$

$$\frac{1}{r}\frac{\partial \rho r u}{\partial r} + \frac{1}{r}\frac{\partial \rho v}{\partial \vartheta} + \frac{\partial \rho r w}{\partial z} = 0 \quad (4)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial r} + \frac{v}{r}\frac{\partial \theta}{\partial \vartheta} + w\frac{\partial \theta}{\partial z} = \dot{\theta} \quad (5)$$

Equations

Exner–function

$$\pi \equiv \left(\frac{p}{p_o}\right)^{R/c_p}, \quad T = \pi\theta \quad (6)$$

Hydrostatic approximation:

$$\frac{\partial p}{\partial z} = -\rho g \overset{\text{ideal gas}}{\Leftrightarrow} \frac{d\ \log \pi}{dz} = \frac{g}{c_p T} \quad (7)$$

Absolute angular momentum M:

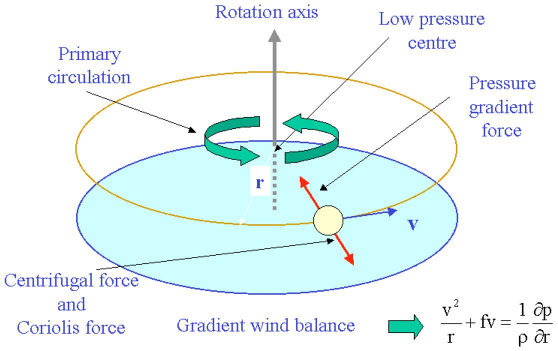
$$M \equiv rv + \frac{1}{2}fr^2 \quad (8)$$

Equation for evolution of M

$$\frac{\partial M}{\partial t} + u\frac{\partial M}{\partial r} + \frac{v}{r}\frac{\partial M}{\partial \vartheta} + w\frac{\partial M}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial \vartheta} \quad (9)$$

For an axisymmetric flow without friction: $-\frac{1}{\rho}\frac{\partial p}{\partial \vartheta} = 0$

Primary circulation



Smith, 2006, pers. comm.

Primary circulation

Steady flow $\frac{\partial}{\partial t} = 0$, axisymmetric $\frac{\partial}{\partial \vartheta} = 0$, no secondary circulation $u = 0$, hydrostatic balance $\frac{\partial p}{\partial z} = -g\rho$
this leads to gradient wind balance

$$\underbrace{\frac{v^2}{r}}_{\text{centrifugal}} + \underbrace{fv}_{\text{Coriolis}} = \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{10}$$

Thermal wind balance using $C \equiv \frac{v^2}{r} + fv$:

$$g\frac{\partial \log \rho}{\partial r} + C\frac{\partial \log \rho}{\partial z} = -\frac{\partial C}{\partial z} \tag{11}$$

Linear partial differential equation (PDE)

Linear PDEs - method of characteristics

Consider linear PDE

$$a(x,y)\frac{\partial u}{\partial x} + b(x,y)\frac{\partial u}{\partial y} = c(x,y) \tag{12}$$

Find (3D) paths $c(s) = (x(s), y(s), z(s))$ along which the PDE degenerates to a system of ordinary differential equations (ODEs):

$$\frac{dx}{ds} = a(x(s), y(s)) \tag{13}$$

$$\frac{dy}{ds} = b(x(s), y(s)) \tag{14}$$

$$\frac{dz}{ds} = c(x(s), y(s)) \tag{15}$$

This set of equations is known as characteristic equations, the path usually is called characteristics or charaacteristic path/line

- Maybe this system of ODEs can be solved
- Qualitative insights can be derived from the characteristics

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Linear PDEs

Example: advection equation

$$a\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0 \tag{16}$$

$u = u(x(s), y(s))$ along path with parameter

$$\frac{du}{ds} = \frac{\partial u}{\partial x}\frac{dx}{ds} + \frac{\partial u}{\partial t}\frac{dt}{ds} \tag{17}$$

with $\frac{dx}{ds} = a$ and $\frac{dt}{ds} = 1$, this results into the original PDE

$$\frac{du}{ds} = a\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0 \Rightarrow u(s) = u_0 = f(x_0) \tag{18}$$

Characteristic equations:

$$\frac{dx}{ds} = a \Rightarrow x(s) = a \cdot s + x_0, \quad \frac{dt}{ds} = 1 \Rightarrow t(s) = s + t_0 \tag{19}$$

$$\frac{dx}{dt} = \frac{\frac{dx}{ds}}{\frac{dt}{ds}} = a \Rightarrow x(t) = a \cdot t + x_0 \tag{20}$$

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Primary circulation

For the linear PDE and the function $f = \log(\rho)$

$$g\frac{\partial \log \rho}{\partial r} + C\frac{\partial \log \rho}{\partial z} = -\frac{\partial C}{\partial z} \Rightarrow g\frac{\partial f}{\partial r} + C\frac{\partial f}{\partial z} = -\frac{\partial C}{\partial z} \tag{21}$$

we use the method of characteristics with $r = r(s), z = z(s)$:

$$\frac{df}{ds} = \frac{\partial f}{\partial r}\frac{dr}{ds} + \frac{\partial f}{\partial z}\frac{dz}{ds} = -\frac{\partial C}{\partial z} \tag{22}$$

this leads to

$$\frac{dr}{ds} = g; \frac{dz}{ds} = C \ \Rightarrow \frac{dz}{dr} = \frac{\frac{dz}{ds}}{\frac{dr}{ds}} = \frac{C}{g} \tag{23}$$

for the characteristic path/line

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Primary circulation

For a small displacement along isobaric surfaces:

$$\frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz = 0 \tag{24}$$

using the hydrostatic balance equation and the gradient wind balance equation we can derive:

$$\frac{dz}{dr} = \frac{C}{g} \text{ along the isobars} \tag{25}$$

i.e. characteristic paths = isobars

From this equation we can estimate the height of the isobars:

$$H(p) = \int_0^{r_p} \frac{C(r)}{g} dr \tag{26}$$

Primary circulation

The density variation (along this equation, i.e. along isobars) can be derived as

$$d\log\rho = -\frac{1}{g}\frac{\partial C}{\partial z}dr = -\frac{1}{g}\left(\frac{2v}{r}+f\right)\frac{\partial v}{\partial z}dr \qquad (27)$$

Interpretations of this result:

1. For a barotropic vortex: $\frac{\partial v}{\partial z} = 0$, hence ρ is constant along isobars
2. For a cyclonic vortex on the Northern hemisphere ($v > 0$) with tangential wind speed that decays with height ($\frac{\partial v}{\partial z} < 0$) the density decreases with decreasing radius along isobars. Thus, virtual and potential temperature T_v, θ increase, i.e. the core of the vortex must be warm (i.e. $\frac{\partial T_v}{\partial r} < 0$)

Prediction of thermal wind equation is consistent with observations.

Warm core of a TC/eye dynamics

Assuming primary circulation is in gradient wind balance, integrating eq. 10 along r leads to

$$\int_{p(r=0,z)}^{p(r=\infty,z)} dp = p(r=\infty,z) - p(r=0,z) = \int_0^\infty \rho C\, dr \qquad (28)$$

and using the environmental pressure $p_e(z) = p(r=\infty,z)$ one derives

$$p(z) = p_e(z) - \int_0^\infty \rho C\, dr \qquad (29)$$

and the equation for the pressure perturbation:

$$p^* = p - p_e = -\int_0^{r_o} \rho C dr \qquad (30)$$

Warm core of a TC/eye dynamics

Derivative of equation for pressure perturbation in z–direction is then:

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\int_0^{r_o}\rho C dr = \frac{1}{\rho}\int_0^{r_o}\left(\frac{\partial\rho}{\partial z}C + \rho\frac{\partial C}{\partial z}\right) \qquad (31)$$

If we assume a TC in the Northern Hemisphere (i.e. $v > 0$, $\frac{\partial v}{\partial z} < 0$, $f > 0$) we can estimate the integrant:

$$\underbrace{\frac{\partial\rho}{\partial z}}_{\le 0}\underbrace{\left(\frac{v^2}{r}+fv\right)}_{>0} + \underbrace{\rho}_{>0}\underbrace{\left(\frac{2v}{r}+f\right)}_{>0}\underbrace{\frac{\partial v}{\partial z}}_{<0} < 0 \qquad (32)$$

thus, $\frac{\partial}{\partial z}\int_0^{r_o}\rho C dr < 0$ and this induces a subsidence ($dw/dt < 0$).

Warm core of a TC/eye dynamics

This must be consistent with the first law of thermodynamics

$$\frac{D\theta}{Dt} = \dot{\theta} \tag{33}$$

Using the approximation

$$B \approx g\frac{\theta - \theta_a}{\overline{\theta}} \tag{34}$$

we can express the first law as:

$$\frac{DB}{Dt} = \frac{g}{\overline{\theta}}\left(\frac{D\theta}{Dt} - \frac{D\theta_e}{Dt}\right) = \underbrace{\frac{g}{\overline{\theta}}\dot{\theta}}_{=S_\theta} - \frac{g}{\overline{\theta}}\frac{\partial\theta_a}{\partial z}w = S_\theta - N^2 w \tag{35}$$

Warm core of a TC

This can be translated into the following equation:

$$\frac{DB}{Dt} = \frac{\partial B}{\partial t} + w\frac{\partial B}{\partial z} = S_\theta - N^2 w \Leftrightarrow w\underbrace{\left(\frac{\partial B}{\partial z} + N^2\right)}_{=N^2_{eye}} = S_\theta - \frac{\partial B}{\partial t} \tag{36}$$

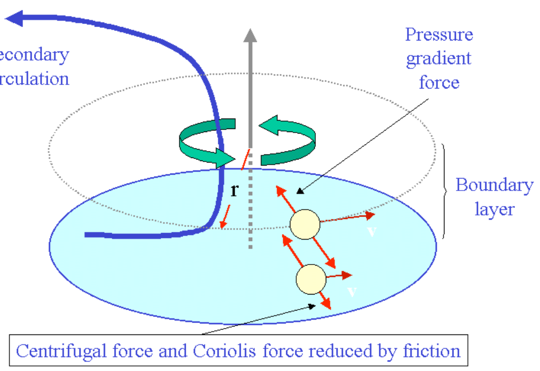
We can assume, that the eye of a TC is nearly in hydrostatic balance ($\frac{\partial p}{\partial z} = -g\rho$)

Interpretation:

The rate of subsidence is that required to warm the air to the degree that its buoyancy remains close to the hydrostatic balance with the pressure gradient

The vertical velocity cannot be estimated

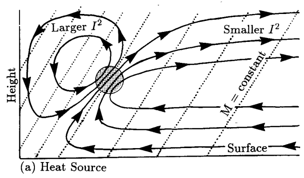
Secondary circulation



Smith, 2006, pers. comm.

Secondary circulation

Additional source for subsidence inside the eye region:

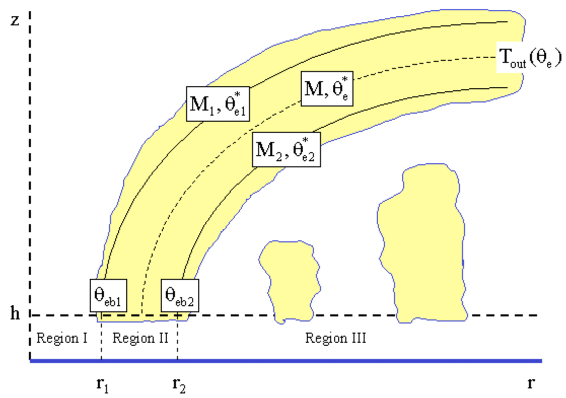


Willoughby, 1995

External heat source drives a strong upward motion but also a weak downward motion.

Heat source = latent heat release from condensation

Steady state – Emanuel model



after Emanuel, 1986

Carnot cycle

Classical Carnot cycle in thermodynamics:

1. Isothermal expansion (gas is been heated, pressure reduced), i.e. θ_e increases
2. Adiabatic expansion (no heat exchange, pressure reduced), i.e. θ_e constant
3. Isothermal compression (gas is been cooled, pressure increased), i.e. θ_e decreases
4. Adiabatic compression (no heat exchange, pressure increased), i.e. θ_e constant

The (mechanical) work from the Carnot engine can be determined by

$$W = Q\left(\frac{T_{hot} - T_{cold}}{T_{hot}}\right) \qquad (37)$$

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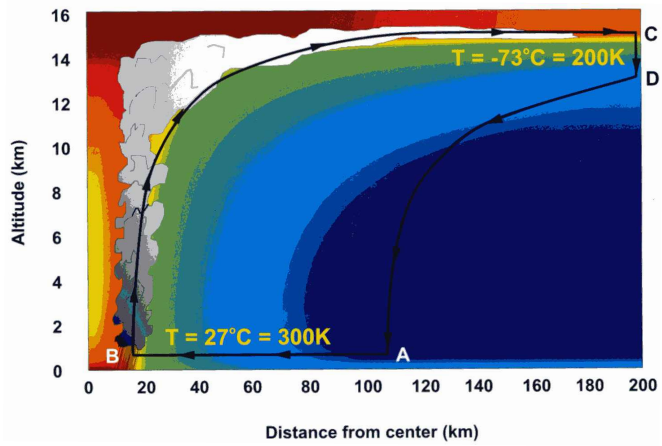
TC as a Carnot cycle



Emanuel, 2005

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TC as a Carnot cycle

Interpretation:

- A to B: Air acquires moisture from the ocean by evaporation of sea water (θ_e increases)
- B to C: Air ascends in the eye wall without acquiring/loosing heat other than by condensation (θ_e constant)
- C to D: Air looses heat (originated from the ocean) by radiative cooling (θ_e decreases)
- D to A: Air descends until it reaches the starting point far outside from the TC (θ_e constant)

Question: What about the mechanical work released in this cycle?

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TC as a Carnot cycle

Some thermodynamics: saturated moist entropy s^* defined as

$$T ds^* = c_v dT + p d\alpha + L dq_{vs}, \;\; h \equiv c_v T + p\alpha + L q_{vs} \qquad (38)$$

From the definition of the saturation equivalent potential temperature $\theta_{es} = \theta \exp((Lq_{vs})/(c_p T))$ follows:

$$c_p T d\log\theta_{es} = \underbrace{c_p T d\log\theta + L dq_{vs}}_{=Tds^*} \underbrace{-LT^{-1}dT}_{negligible} \approx T ds^* \qquad (39)$$

$$\Delta Q_{AB} = \int_{s_a^*}^{s^*} T_B ds^* = \int_{\theta_{ea}}^{\theta_e} c_p T_B d\log\theta_e = c_p T_B \log\left(\frac{\theta_e}{\theta_{ea}}\right) \qquad (40)$$

$$\Delta Q_{CD} = \int_{\theta_e}^{\theta_{ea}} c_p T_{out} d\log\theta_e = -c_p T_{out}\log\left(\frac{\theta_e}{\theta_{ea}}\right) \qquad (41)$$

$$\Delta Q = \Delta Q_{AB} + \Delta Q_{CD} = c_p T_B \epsilon \log\left(\frac{\theta_e}{\theta_{ea}}\right); \;\; \epsilon = \frac{T_B - T_{out}}{T_B} \qquad (42)$$

TC as a Carnot cycle

We have derived:

$$W = \Delta Q = \epsilon T_B \Delta s^* \qquad (43)$$

In equilibrium this energy production must equal the dissipation of the system (i.e. mainly due to friction in the boundary).

TC as a Carnot cycle

Assumptions:

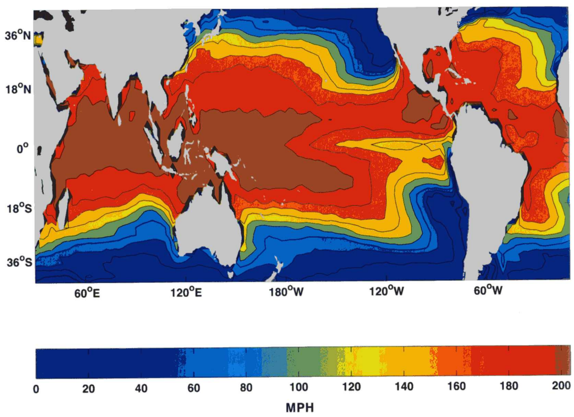
- entropy is added to the atmosphere by the sea
- momentum is lost to the sea
- integration only to the boundary of the eyewall (i.e. radius with maximal wind speed)

$$\int_{r_m}^{r_o} \rho \epsilon T_B C_k |\vec{V}| (s_S^* - s_B) r dr = \int_{r_m}^{r_o} \rho C_D |\vec{V}|^3 r dr \qquad (44)$$

C_K denotes the coefficient controlling enthalpy fluxes
C_D denotes the drag coefficient
Assumption: Largest contribution for the integrals comes from the flow near the radius of maximal wind speed. Thus, the equation can be written as:

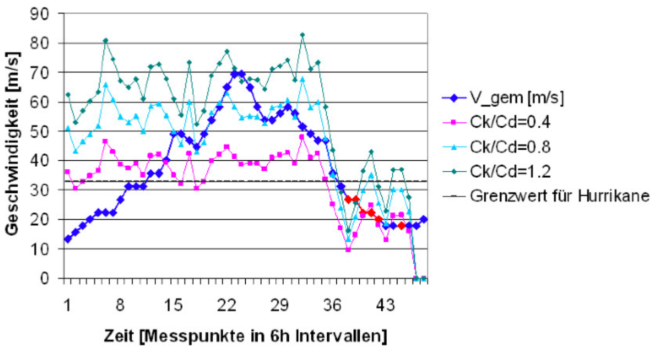
$$|\vec{V}_m|^2 \approx \frac{C_k}{C_D} \epsilon T_B (s_S^* - s_B) \Big|_m \qquad (45)$$

Maximal wind speeds



Emanuel, 2005

Comparison



Hurricane Floyd, 1999, Category 4

R. Lorenz, Bachelor thesis, 2007

TC as a Carnot cycle

Estimation of the central pressure deviation from the maximum wind speed:

Assumption: Rankine vortex for cyclostrophic balance

$$v(r) = v_m \frac{r}{r_m} \text{ for } r \leq r_m \tag{46}$$

this yields the following equation:

$$v_m^2 \frac{r}{r_m^2} = \frac{v^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{RT}{p}\frac{\partial p}{\partial r} = RT\frac{\partial \log p}{\partial r} \tag{47}$$

By separation of the variables and integration:

$$\int_0^{r_m} v_m^2 \frac{r}{r_m^2} dr = \int_{\log p_c}^{\log p_m} RTd \log p \tag{48}$$

this leads to:

$$p_c = p_m \exp\left(-\frac{v_m^2}{2RT_B}\right) \tag{49}$$

Summary of first part

- Primary circulation of TC's can be described by gradient wind balance
- In the core of TC's subsidence can be found (warm core), this is consistent with the primary circulation
- TC's can be described using the Carnot process:
  - Heating due to moisture from the boundary layer
  - Maximal tangential wind speed can be estimated from the thermodynamic properties
  - Pressure deviation in the core of TC's can be estimated from the maximal tangential wind speed

Next lectures:

- Steady state model (Emanuel, 1986) in details (tangential wind, temperature and momentum distributions)
- Tracks of hurricanes
- Models and forecasts