② ⓐ

1) $w(x,y,z) = w_0 \exp(-a(x^2+y^2))$

2) $$\nabla_h w = \begin{pmatrix} \frac{\partial w(x,y,z)}{\partial x} \\ \frac{\partial w(x,y,z)}{\partial y} \end{pmatrix} = \begin{pmatrix} -2aw_0 x \exp(-a(x^2+y^2)) \\ -2aw_0 y \exp(-a(x^2+y^2)) \end{pmatrix}$$

$$= -2aw_0 \exp(-a(x^2+y^2)) \begin{pmatrix} x \\ y \end{pmatrix}$$

3.a) $$\vec{S} = \frac{\partial \vec{V}}{\partial z} = \frac{\partial}{\partial z} \begin{pmatrix} U_0 + S_x z \\ 0 \end{pmatrix} = \begin{pmatrix} S_x \\ 0 \end{pmatrix}$$

4.a) $p^* \approx C \cdot 2 \rho_0 \vec{S} \nabla_h w$

$$p^* \approx C \cdot 2\rho_0 \begin{pmatrix} S_x \\ 0 \end{pmatrix} \left[-2aw_0 \exp(-a(x^2+y^2)) \right] \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= -C \cdot 2\rho_0 2 a w_0 \exp(-a(x^2+y^2)) \left[S_x x + 0 \cdot y \right]$$

$$= \underbrace{-4C\rho_0 a w_0 S_x}\exp(-a(x^2+y^2))\, x$$

$$= -4 \cdot 10^6 m \cdot 1 \frac{kg}{m^3} \cdot \frac{1}{2} 10^{-6} \frac{1}{m^2} \cdot 20 \frac{m}{s} 10^{-2} \frac{1}{s}$$

$$= -0{,}4 \frac{kg}{m^3 s^2}$$

$$= -0{,}4 \frac{kg}{m^3 s^2} \exp\left(-0{,}5 \cdot 10^{-6} \frac{1}{m^2} (x^2+y^2)\right) \cdot x$$

Units:

$$\frac{kg}{m^3 s^2} \cdot m = \frac{kg\,m}{s^2} \frac{1}{m^3} = \frac{N}{m^3} = Pa \checkmark$$

$$= -0{,}4 \frac{kg}{m^3 s^2} \exp\left(-0{,}5 \cdot 10^{-6} \frac{1}{m^2} \cdot 1000^2 m^2\right) \cdot x$$

$$= -0{,}243 \frac{kg}{m^3 s^2} x$$

Values in table 1. Figure an interpretation after the table

② ⓑ

3.b) $$\vec{S} = \frac{\partial \vec{V}}{\partial z} = -V_0 \frac{\partial}{\partial z}\left[\begin{pmatrix} \cos\left(2\pi - \pi \frac{z}{z_0}\right) \\ \sin\left(2\pi - \pi \frac{z}{z_0}\right)\end{pmatrix}\right]$$

$$= V_0 \frac{\pi}{z_0}\begin{pmatrix} \sin\left(2\pi - \pi \frac{z}{z_0}\right) \\ \cos\left(2\pi - \pi \frac{z}{z_0}\right)\end{pmatrix}$$

4.b) $$p^* \approx C \cdot 2 \cdot \rho_0 \vec{S} \nabla_h w$$

$$= C 2 \rho_0 V_0 \frac{\pi}{z_0}\begin{pmatrix} \sin\left(2\pi - \pi \frac{z}{z_0}\right) \\ \cos\left(2\pi - \pi \frac{z}{z_0}\right)\end{pmatrix}\left[-2 a w_0 \exp\left(-a(x^2+y^2)\right)\begin{pmatrix} x \\ y \end{pmatrix}\right]$$

$$= \underbrace{-C 2 \rho_0 V_0 \frac{\pi}{z_0} 2 a w_0 \exp\left(-a\, 1000^2\right)}_{= -0{,}381 \frac{kg}{m^3 s^2}} \cdot \left[\sin\left(2\pi - \pi \frac{z}{z_0}\right) x + \cos\left(2\pi - \pi \frac{z}{z_0}\right) y\right]$$

$$= -0{,}381 \frac{kg}{m^3 s^2}\left[x \begin{cases} z = 0 & : 0 \\ z = 2000 : & -1 \\ z = 4000 : & 0 \end{cases} + y \begin{cases} z = 0 & : 1 \\ z = 2000 : & 0 \\ z = 4000 : & -1 \end{cases}\right]$$

Units:

$$\left[\frac{V_0}{z_0}\right] = [S_x] \checkmark$$

Values in table 1. Figure an interpretation after the table

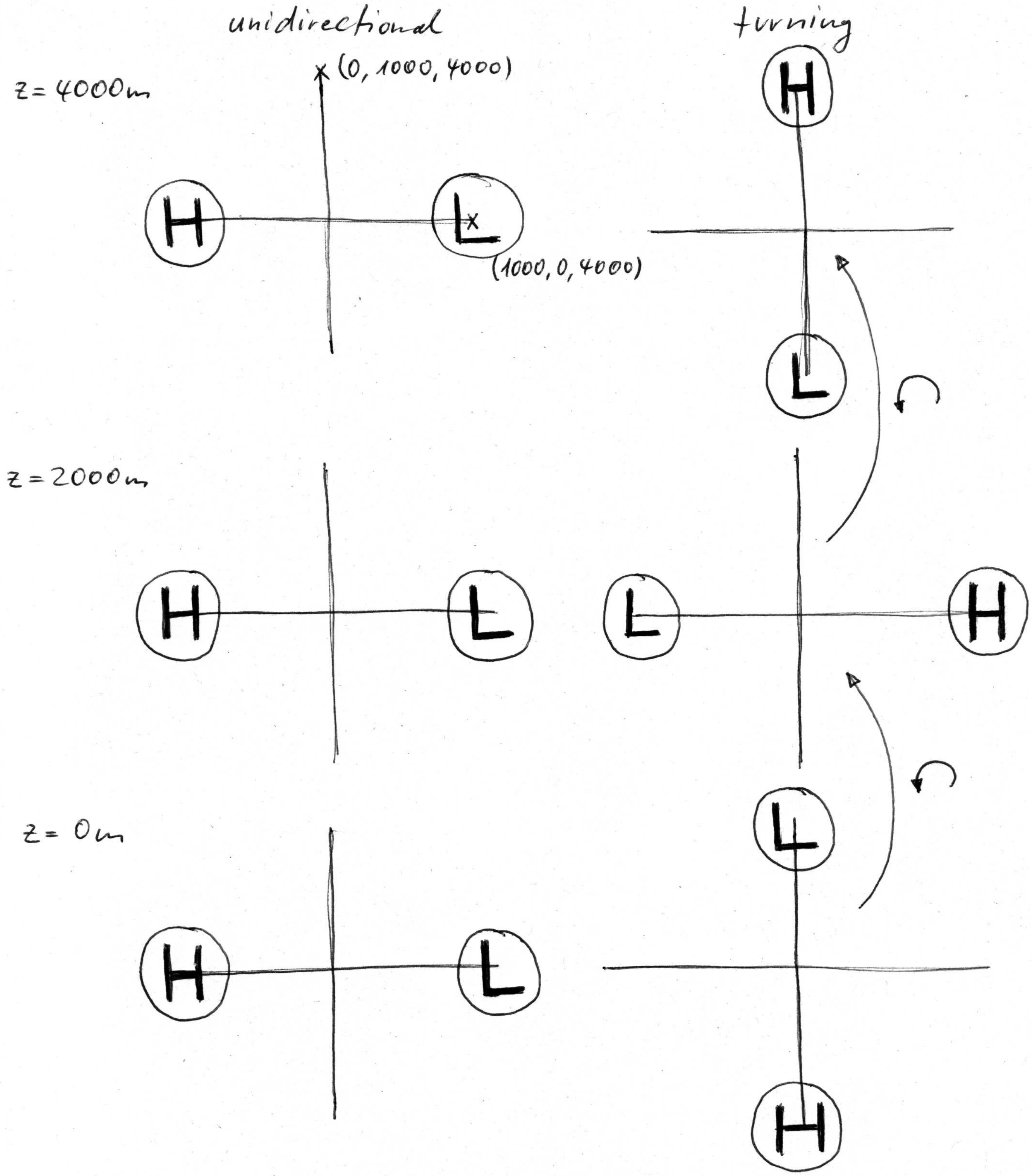
② Table 1

Table 1: p^* for unidirectional and turning wind shear

Position [m]	p^* (unidirectional 4.a)) [Pa]	p^* (turning 4.b)) [Pa]
$\vec{x}_1 = (1000, 0, 0)$	$-242{,}6$	0
$\vec{x}_2 = (-1000, 0, 0)$	$+242{,}6$	0
$\vec{x}_3 = (0, 1000, 0)$	0	$-381{,}1$
$\vec{x}_4 = (0, -1000, 0)$	0	$+381{,}1$
$\vec{x}_5 = (1000, 0, 2000)$	$-242{,}6$	$+381{,}1$
$\vec{x}_6 = (-1000, 0, 2000)$	$+242{,}6$	$-381{,}1$
$\vec{x}_7 = (0, 1000, 2000)$	0	0
$\vec{x}_8 = (0, -1000, 2000)$	0	0
$\vec{x}_9 = (1000, 0, 4000)$	$-242{,}6$	0
$\vec{x}_{10} = (-1000, 0, 4000)$	$+242{,}6$	0
$\vec{x}_{11} = (0, 1000, 4000)$	0	$+381{,}1$
$\vec{x}_{12} = (0, -1000, 4000)$	0	$-381{,}1$

② Figure

wind shear



(3) i)

$$0)\quad v_{max}^2 = \frac{C_K}{C_p}\,\varepsilon\, T_s \left[\ln\left(\theta_{es}^*\right) - \ln\left(\theta_e\right)\right]$$

$$1)\quad \frac{C_K}{C_p} = 1{,}2$$

$$2)\quad \varepsilon = \frac{T_B - T_0}{T_0} = \frac{T_s - T_0}{T_0} = \frac{\begin{cases}300\,K\\302\,K\\305\,K\end{cases} - 206\,K}{206\,K} = \underline{\begin{cases}0{,}456\\0{,}466\\0{,}481\end{cases}}$$

$$3)\quad T_s = \underline{\begin{cases}300\,K\\302\,K\\305\,K\end{cases}}$$

$$4)\quad \theta_{es}^* = \theta \exp\left(\frac{L\, q_{sat}(T_s)}{c_p\, T_s}\right)$$

$$5)\quad \theta = \underbrace{T_B}_{=T_s}\left(\frac{\hat{p}}{\underbrace{p}_{=p_B=p_s}}\right)^{\frac{R}{c_p}} = \begin{cases}300\,K\\302\,K\\305\,K\end{cases}\left(\frac{10^5\,Pa}{10^5\,Pa - 2000\,Pa}\right)^{\frac{\cancelto{1}{287{,}04\,\frac{J}{kg\,K}}}{3{,}5\cdot\cancel{287{,}04\,\frac{J}{kg\,K}}}} = \underline{\begin{cases}301{,}74\,K\\303{,}75\,K\\306{,}77\,K\end{cases}}$$

$$6)\quad RH = 100\%\,\frac{p\,q}{\varepsilon_0\, p_{sat}(T_s)}$$

$$q = \frac{RH\,\varepsilon_0\,p_{sat}(T_s)}{100\%\,p_s} = \frac{75\%\cdot 0{,}622\cdot\begin{cases}3536{,}76\,Pa\\3974{,}42\,Pa\\4719{,}13\,Pa\end{cases}}{100\%\,(10^5 - 2000)\,Pa} = \underline{\begin{cases}1{,}68\cdot 10^{-2}\,kg/kg\\1{,}89\cdot 10^{-2}\,kg/kg\\2{,}25\cdot 10^{-2}\,kg/kg\end{cases}}$$

$$7)\quad q_{sat} = \frac{\varepsilon_0\,p_{sat}(T_s)}{p_s} = \frac{q}{0{,}75} = \frac{\begin{cases}1{,}68\cdot 10^{-2}\,kg/kg\\1{,}89\cdot 10^{-2}\,kg/kg\\2{,}25\cdot 10^{-2}\,kg/kg\end{cases}}{0{,}75} = \underline{\begin{cases}2{,}24\cdot 10^{-2}\,kg/kg\\2{,}52\cdot 10^{-2}\,kg/kg\\3{,}00\cdot 10^{-2}\,kg/kg\end{cases}}$$

diese Werte sind noch im Stack meines TR ☺

$$4.1)\quad \theta_{es}^* = \begin{cases}301{,}74\,K\\303{,}75\,K\\306{,}77\,K\end{cases}\exp\frac{2{,}53\cdot 10^6\,\frac{J}{kg}\begin{cases}2{,}24\cdot 10^{-2}\,kg/kg\\2{,}52\cdot 10^{-2}\,kg/kg\\3{,}00\cdot 10^{-2}\,kg/kg\end{cases}}{3{,}5\cdot 287{,}04\,\frac{J}{kg\,K}\begin{cases}300\,K\\302\,K\\305\,K\end{cases}} = \underline{\begin{cases}364{,}30\,K\\374{,}86\,K\\392{,}84\,K\end{cases}}$$

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8) $\theta_e = \theta \exp\left(\dfrac{Lq}{c_{p_{w}} \underbrace{T}_{=T_B=T_S}}\right)$

$$= \begin{cases} 301{,}74\,K \\ 303{,}75\,K \\ 306{,}77\,K \end{cases} \exp\left(\dfrac{2{,}53\cdot 10^6 \frac{J}{kg} \cdot \begin{cases} 1{,}68\cdot 10^{-2}\,kg/kg \\ 1{,}89\cdot 10^{-2}\,kg/kg \\ 2{,}25\cdot 10^{-2}\,kg/kg \end{cases}}{3{,}5\cdot 287{,}04\,\frac{J}{kg\,K}\cdot \begin{cases} 300\,K \\ 302\,K \\ 305\,K \end{cases}}\right) = \underline{\begin{cases} 347{,}54\,K \\ 355{,}66\,K \\ 369{,}28\,K \end{cases}}$$

0,1) $$v_{max}^2 = \underbrace{\frac{C_K}{C_D}\cdot c_p}_{=1205{,}57\,\frac{J}{kg\,K}} \cdot \begin{cases} 0{,}456 \\ 0{,}466 \\ 0{,}481 \end{cases} \cdot \begin{cases} 300\,K \\ 302\,K \\ 305\,K \end{cases} \cdot \left[\ln\left(\begin{cases} 364{,}30\,K \\ 374{,}86\,K \\ 392{,}84\,K \end{cases}\right) - \ln\left(\begin{cases} 347{,}54\,K \\ 355{,}66\,K \\ 369{,}28\,K \end{cases}\right)\right] = \begin{cases} 88{,}17^2\,\frac{m^2}{s^2} \\ 94{,}46^2\,\frac{m^2}{s^2} \\ 104{,}52\,\frac{m^2}{s^2} \end{cases}$$

$$\boxed{\begin{gathered}\text{Units:}\\ \frac{J\,\cancel{K}}{kg\,\cancel{K}} = \frac{Nm}{kg} = \frac{\cancel{kg}\,m\,m}{s^2\,\cancel{kg}} = \frac{m^2}{s^2}\ \checkmark\end{gathered}}$$

0,2) $$v_{max} = \underline{\underline{\begin{cases} 88\,m/s & \leftarrow 300K = T_s \\ 94\,m/s & \leftarrow 302K = T_s \\ 105\,m/s & \leftarrow 305K = T_s \end{cases}}}$$

3 ii) Equation (49) from the lecture:

0) $$p_c = p_{max} \exp\left(-\frac{V_{max}^2}{2RT_B}\right)$$

$$= (10^5 - 2000)\,Pa \exp\left(-\frac{\begin{cases} 88{,}17^2 \frac{m^2}{s^2} \\ 94{,}46^2 \frac{m^2}{s^2} \\ 104{,}52 \frac{m^2}{s^2} \end{cases}}{2 \cdot 287{,}04 \frac{J}{kg K} \begin{cases} 300 K \\ 302 K \\ 305 K \end{cases}}\right)$$

$$= \begin{cases} 93674\,Pa \\ 93084\,Pa \\ 92072\,Pa \end{cases} = \begin{cases} 937\,hPa \\ 931\,hPa \\ 921\,hPa \end{cases}$$

Pressure deviation:

1) $$p_{max} - p_c = (10^5 - 2000)Pa - \begin{cases} 93674\,Pa \\ 93084\,Pa \\ 92072\,Pa \end{cases}$$

$$= \begin{cases} 43{,}26\,hPa \\ 49{,}16\,hPa \\ 59{,}28\,hPa \end{cases}$$