Cloud Dynamics, Assigment #3

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1 Lightning and hurricanes

Is lightning and thunder as comman in hurricanes as in extra-tropical tunderstorms?
A hurricane research campaign (of the NASA) in 1998 showed that "hurricanes typically don't produce much lightning, but sometimes they can." Some longtime hurricane pilots have reported that when a hurricane does produce lightning, intensification often follows.
The third Convection and Moisture Experiment (CAMEX 3) in August and September 1998 focused a number of ground, airborne, and satellite instruments on hurricane activities in the western Atlantic Ocean, the Gulf of Mexico and on thunderstorms over Florida. The ER-2 (a reconnaissance aircraft, equipped with eight lightning detectors among other instruments) recorded only a few lightning flashes as it flew over the eye of hurricane Bonnie on Aug. 26. 1998.
Hurricane Georges was a different matter when it waded ashore and battered the island of Hispaniola on Sept. 22.1998 Georges showed nearly continuous lightning, in and around the eye. Bateman from the Universities Space Research Association said that the increased lightning around Georges' eye probably was due to air forced upward—called orographic forcing—when the hurricane hit the mountains.
Mr Blakeslee of the GHCC[1] agrees with that, when he says: "Hurricanes are most likely to produce lightning when they're making landfall."
This seems to be a reasonable explanation. But the "orographic forcing" can not be the only reason, as measurements from the year 2005 show. There were

[1] Global Hydrology and Climate Center in Huntsville, Alabama

no mountains beneath the "electric hurricanes" (Emily, Rita and Katrina) of 2005-only flat water.

The NASA describes the source of lightening as follows: "Within thunder-clouds, vertical winds cause ice crystals and water droplets (called "hydrom-eteors") to bump together. This "rubbing" causes the hydrometeors to become charged. For reasons not fully understood, positive electric charge accumulates on smaller particles while negative charge clings to the larger ones. Winds and gravity separate the charged hydrometeors, producing an enormous electric field within the storm. This is the source of lightning." A hurricane's winds are mostly horizontal, not vertical. So the vertical churning that leads to lightning doesn't normally happen.

According to the NASA measurements the three "electric hurricanes" (Emily, Rita and Katrina) in the year 2005 had a few similarities: all three storms were powerful: Emily was a Category 4 storm, Rita and Katrina were Category 5 all three were over water when their lightning was detected in each case, the lightning was located around the eye-wall.

Due to this fact, one possible explanation for the lightnings in the hurricanes could be the big power of these hurricanes, so that sheer violence somehow causes their lightning. But for Blakeslee this explanation is too simple: "Other storms have been equally intense and did not produce much lightning. There must be something else at work."

Another reasonable theory could be, that the vertical winds somehow turn into horizontal ones (for e.g. as they do in normal thunderstorms)—due to orographic forcing, convective forces or other reasons. Once there is a little vertical up- and downdraft, this could be reinforced, get stronger and lead to lightning. This would at least explain the fact, why they usually do hardly have lightning, and suddenly they have an extreme one.[2]

[2] Sources: http://science.nasa.gov/newhome/headlines/essd16jun99_1.htm, http://science.nasa.gov/headlines/y2006/09jan_electrichurricanes.htm and http://www.nasa.gov/mission_pages/hurricanes/archives/2006/hurricane_lightning.html

2 Storm surges

2.1 Wind driven surge

$$\Delta\zeta = \Delta x \frac{\tau_0}{\varrho g H} \tag{1}$$

with:

$$\begin{aligned}
\Delta x &= 100\,\mathrm{km} = 10^5\,\mathrm{m} \\
\tau_0 &= 2.6 \cdot 10^{-3} 1.25\,\mathrm{kg\,m^{-3}}(40\,\mathrm{m\,s^{-1}})^2 = 5.2\,\mathrm{kg\,m^{-1}\,s^2} \\
\varrho &= 1000\,\mathrm{kg\,m^{-3}} \\
g &= 9.81\,\mathrm{m\,s^{-2}} \\
H &= 20\,\mathrm{m}
\end{aligned}$$

follows

$$\Delta\zeta = 10^5\,\mathrm{m}\frac{5.2\,\mathrm{kg\,m^{-1}s^{-2}}}{1000\,\mathrm{kg\,m^{-3}} \cdot 9.81\,\mathrm{m\,s^{-2}} \cdot 20\,\mathrm{m}} \tag{2}$$

$$= 2.65\,\mathrm{m} \tag{3}$$

2.2 Pressure surge

Lift up the water with the low pressure

$$\Delta p = \varrho g h_p \tag{4}$$

$$h_p = \frac{\Delta p}{\varrho g} \tag{5}$$

For example, Hurricane Juan had a central pressure of 974 hPa when it made landfall just west of Halifax Harbour. Δp is approximately 40 hPa (1013 hPa-975 hPa).

$$h_p = \frac{4000\,\mathrm{Pa}}{1000\,\mathrm{kg\,m^{-3}} \cdot 9.81\,\mathrm{m\,s^{-2}}} = 0.41\,\mathrm{m} \tag{6}$$

2.3 Superposition

Add result (3) and (6)

$$
\begin{aligned}
h &= \Delta\zeta + h_p & (7)\\
&= 2.65\,\mathrm{m} + 0.41\,\mathrm{m} & (8)\\
&= 3.06\,\mathrm{m} & (9)
\end{aligned}
$$

The contribution of the pressure surge is one order of magnitude smaller therfore the wind surge is much more important.

3 Buoyancy and entrainment

3.1 Contributors of buoyancy

$$
B \approx g\left(\underbrace{\frac{T^*}{T_0}}_{=\frac{a}{g}} - \underbrace{\frac{p^*}{p_0}}_{=0} + \underbrace{0.61 q_v^*}_{=\frac{b}{g}} \underbrace{-q_H}_{=\frac{c}{g}}\right) \tag{10}
$$

$$
\begin{aligned}
g &= 9.81\,\mathrm{m\,s^{-2}}\\
T^* &= T_{sa} - T_e\\
T_0 &= T_e(p = 650)\\
p^* &= p_s - p_e \Rightarrow \frac{p^*}{p} = 0\\
q_{v_e} &= q_s\\
q_v^* &= q_{v_{sa}} - q_{v_e}
\end{aligned}
$$

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sa**:** if we go up along the saturated adiabts from $h = 650\,\mathrm{hPa}$.

e**:** environmental value

0**:** value at 650 hPa

The Values from the Tephigram

p [hPa]	T_e [°C]	q_{v_e} [g kg^{-1}]	$q_{v_{sa}}$ [g kg^{-1}]
650	-1	5.5	5.5
600	-5	4.4	4.6
500	-14	2.5	2.8
400	-26	1.2	1.2

With equation (10) follows:

p [hPa]	B [m s^{-2}]	a [m s^{-2}]	b [m s^{-2}]	c [m s^{-2}]
650	0	0	0	0
600	-12.54	-4.91	1.2	-8.83
500	-34.5	-9.81	1.8	-26.49
400	-42.18	0	0	-42.18

The hydrometeors (q_H, ice and liquid water) content of the cloud is the most important term of the equation (10).
Temperature change T^* and absolute vapor content change (q^*) is negligible due to the difference of hydrometeors (q_H). The change of buoyancy B is big, if there is a change in q_H this is the case if water condense.

3.2 Importance of entrainment

$$\frac{\mathrm{d}T_c}{\mathrm{d}z} = -\frac{g}{c_p} - \frac{L}{c_p} \cdot \frac{\mathrm{d}q_{vc}}{\mathrm{d}z} + \Lambda_z \left[(T_e - T_c) + \frac{L}{c_p}(q_{ve} - q_{vc})\right] \tag{11}$$

with

$$\Lambda_z = \frac{1}{m}\left(\frac{\mathrm{d}m}{\mathrm{d}z}\right)_\epsilon = \frac{0.5}{z} \tag{12}$$

is

$$\frac{\mathrm{d}T_c}{\mathrm{d}z} = -\frac{g}{c_p} - \frac{L}{c_p} \cdot \frac{\mathrm{d}q_{vc}}{\mathrm{d}z} + \frac{0.5}{z} \left[(T_e - T_c) + \frac{L}{c_p}(q_{ve} - q_{vc})\right] \tag{13}$$

$$\mathrm{d}T_c = -\frac{g}{c_p}\mathrm{d}z - \frac{L}{c_p} \cdot \frac{\mathrm{d}q_{vc}}{\mathrm{d}z}\mathrm{d}z + \frac{0.5}{z} \left[(T_e - T_c) + \frac{L}{c_p}(q_{ve} - q_{vc})\right] \mathrm{d}z \tag{14}$$

$$\Delta T_c = \underbrace{-\frac{g}{c_p}\Delta z}_{=a} \underbrace{-\frac{L}{c_p} \cdot \mathrm{d}q_{vc}}_{=b} \underbrace{+0.5 \ln \frac{z_{i+1}}{z_i} \left[(T_e - T_c) + \frac{L}{c_p}(q_{ve} - q_{vc})\right]}_{=c} \tag{15}$$

For the altitude in meter

$$\Delta z = \frac{RT}{g} \ln \frac{p_i}{p_{i+1}} \qquad \text{with } R = 287.05\,\mathrm{J\,kg^{-1}\,K^{-1}} \tag{16}$$

follows with T_e and h_0:

p [hPa]	T_e [°C]	h [m]	Δz [m]
650	-1	3860	
600	-5	4497	637
500	-14	5927	1430
400	-26	7618	1691

Now ΔT_c can calculate with equation (15).

p [hPa]	ΔT_c [°C]	T_c	a [°C]	b [°C]	c [°C]
650	–	-1	–	–	–
600	-4.0	-5.0	-6.2	2.2	-0.0
500	-9.6	-14.6	-14.0	4.5	-0.1
400	-12.5	-27.1	-16.5	4.0	0.0

Just term a and b are relevant

With the parcel theory the temperature are (go up along the saturated adiabats):

p [hPa]	T_s [°C]	ΔT_s [°C]
650	-1	–
600	-4.5	-3.5
500	-13	-8.5
400	-26	-13

The temperature with entrainment (T_c) is a little bit lower than the viromental temperature (T_e), that one with the parcel theory (T_s) a little bit higher. The real world is more or less between T_e and T_s.

The four values T_c form a straight line in the tephigram. The tree lines (T_e, T_s, T_c) are also plotted under the table 1.

3.3 Calculations

Made with a spreadsheet, see table 1

Table 1: Calculations

											Term						Term			
p	T_e	Delta T_e	T_s	Delta T_s	T*=T_s-T	q_s	q_sa	q*	q_H=q_sa-q_s	B	a	b	c	h	Delta h	Delta T_c	a	b	c	T_c
650	-1		-1		0	5.5	5.5	0	0	0	0	0	0	3860						
600	-5	-4	-4.5	-3.5	0.5	4.4	4.6	0.2	0.9	-12.54	-4.91	1.2	-8.83	4497.06	637.06	-4.01	-6.22	2.24	-0.03	-5.01
500	-14	-9	-13	-8.5	1	2.5	2.8	0.3	2.7	-34.5	-9.81	1.8	-26.49	5926.81	1429.75	-9.58	-13.96	4.48	-0.1	-14.6
400	-26	-12	-26	-13	0	1.2	1.2	0	4.3	-42.18	0	0	-42.18	7617.93	1691.11	-12.53	-16.51	3.98	0	-27.12