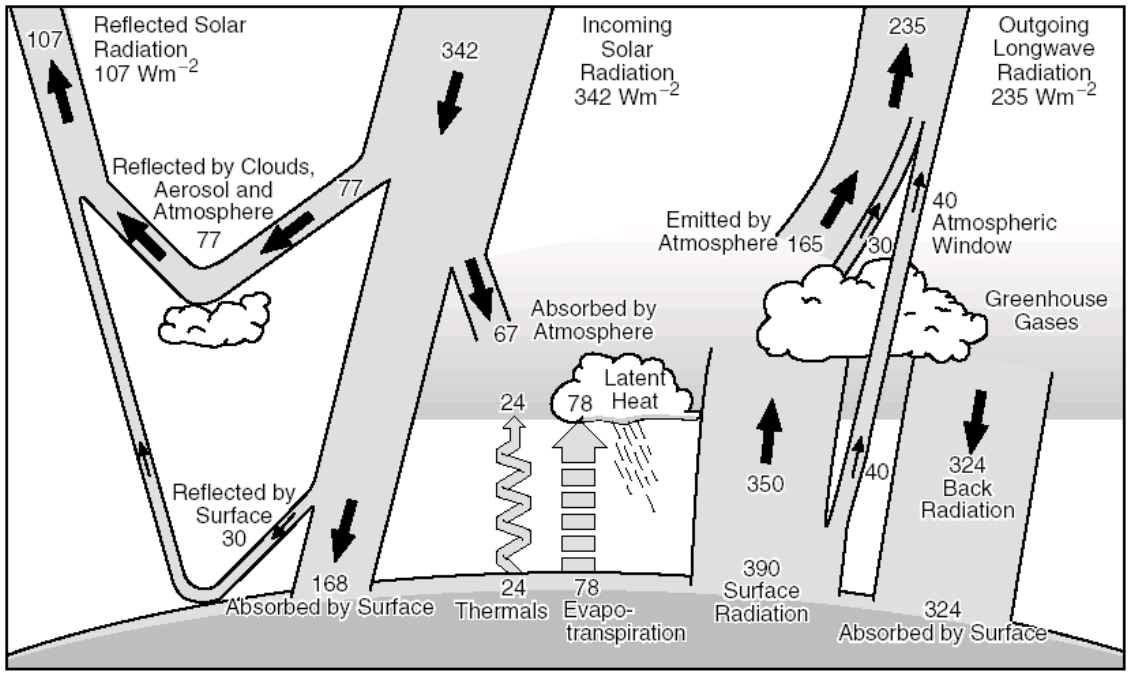
Radiative transfer*

Recall: the global energy balance



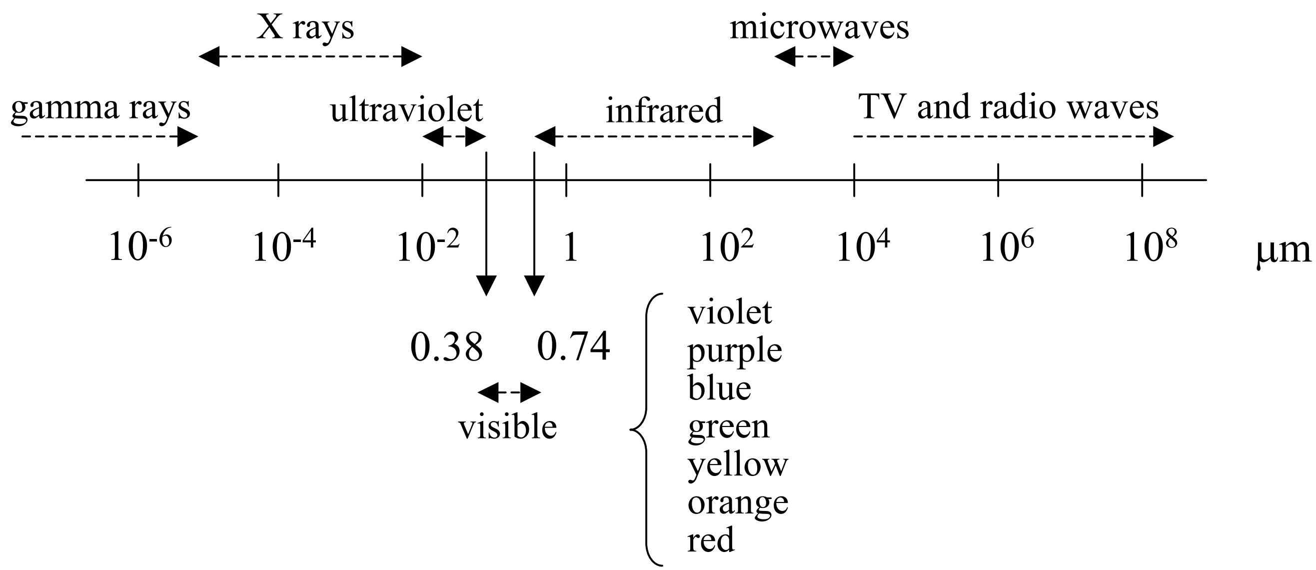
*This chapter is mostly based on (i) Chandrasekhar, S., 1950: Radiative Transfer. Dover, New York; (ii) Liou, K.N., 2002: An Introduction to Atmospheric Radiation (2nd Ed.). Academic Press, Amsterdam; (iii) Paltridge, G.W. and C.M.R. Platt, 1976: Radiative Processes in Meteorology and Climatology, Elsevier, Amsterdam.

Definitions

quantity	definition	symbol	unit
• radiation	electromagnetic energy emitted transferred or received		
• radiant flux	(or radiant power) rate of transfer of radiant energy	$P = dU/dt$	W
• radiant energy	quantity of energy	U J	
• irradiance	radiant flux per unit area incident upon a surface	$F = dP/dA$	$W\ m^{-2}$
• radiance	radiant flux per unit solid angle at a point in a surface per unit projected area of the surface	$N = dF/\cos\theta d\omega$	$W\ m^{-2}\ sr^{-1}$

Electromagnetic waves

The electromagnetic spectrum



Frequency and wave length

$$\lambda = \frac{c}{\nu}$$

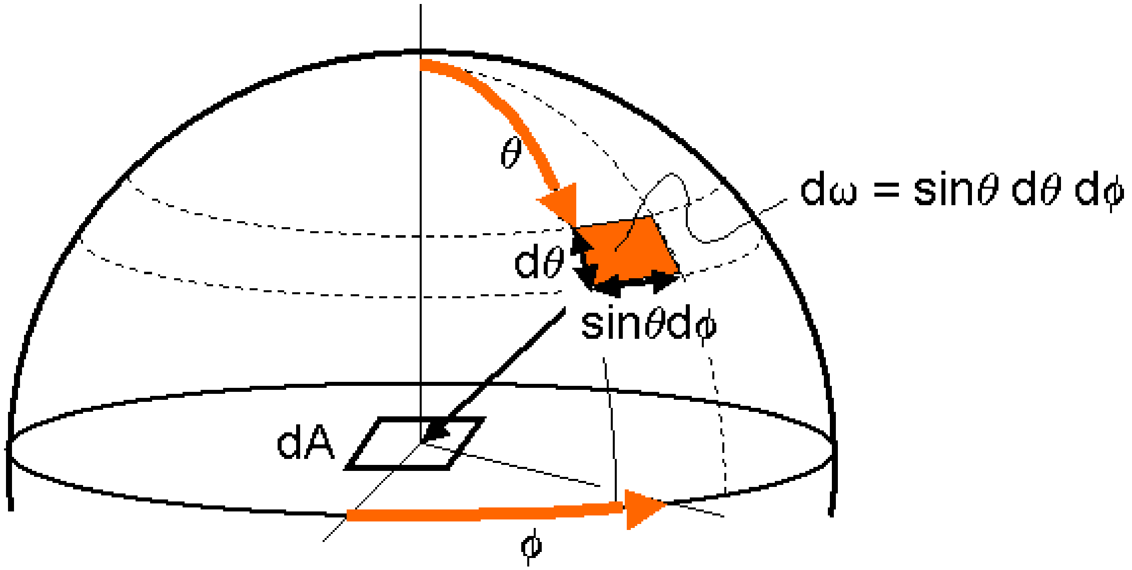
where

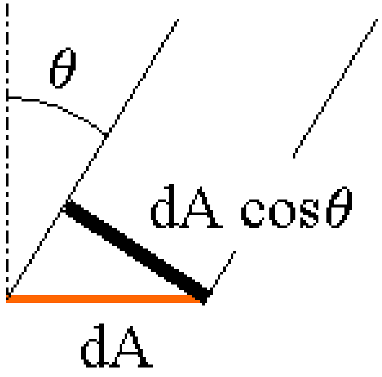
λ = wave length [m]

ν = frequency [s^{-1}]

c = $2.998 \cdot 10^{8}$ m s^{-1}, speed of light in vacuum

Geometrical considerations





Radiance and irradiance

Radiation falling on a surface at an angle θ from the normal to the surface gives at the surface an irradiance confined to the solid angle dω equal to N cosθ dω. The total irradiance at the surface is therefore the integral over the half-sphere of which the surface is the diametral plane:

$$F \equiv \int N \cos\theta \, d\omega$$

In terms of polar co-ordinates θ and ϕ:

$$F \equiv \int_0^{2\pi} d\phi \int_0^{\pi/2} N \cos\theta \sin\theta \, d\theta$$

For isotropic radiation, that is when N is independent of direction, the integral reduces to:

$$F \equiv N \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = \pi N$$

Blackbody radiation

In 1901 Planck was able to derive an analytical expression for the (unpolarized) radiant energy emitted by an enclosure in thermal equilibrium at an absolute temperature T (a blackbody) per unit volume per unit wavelength interval, the so-called Planck function:

$$B_\lambda(T) \equiv 2\,h\,c^2\,\lambda^{-5}\left[\exp\left(\frac{h\,c}{\lambda\,k\,T}\right)-1\right]^{-1} \quad [\mathrm{W\,m^{-2}\,m^{-1}\,sr^{-1}}]$$

where

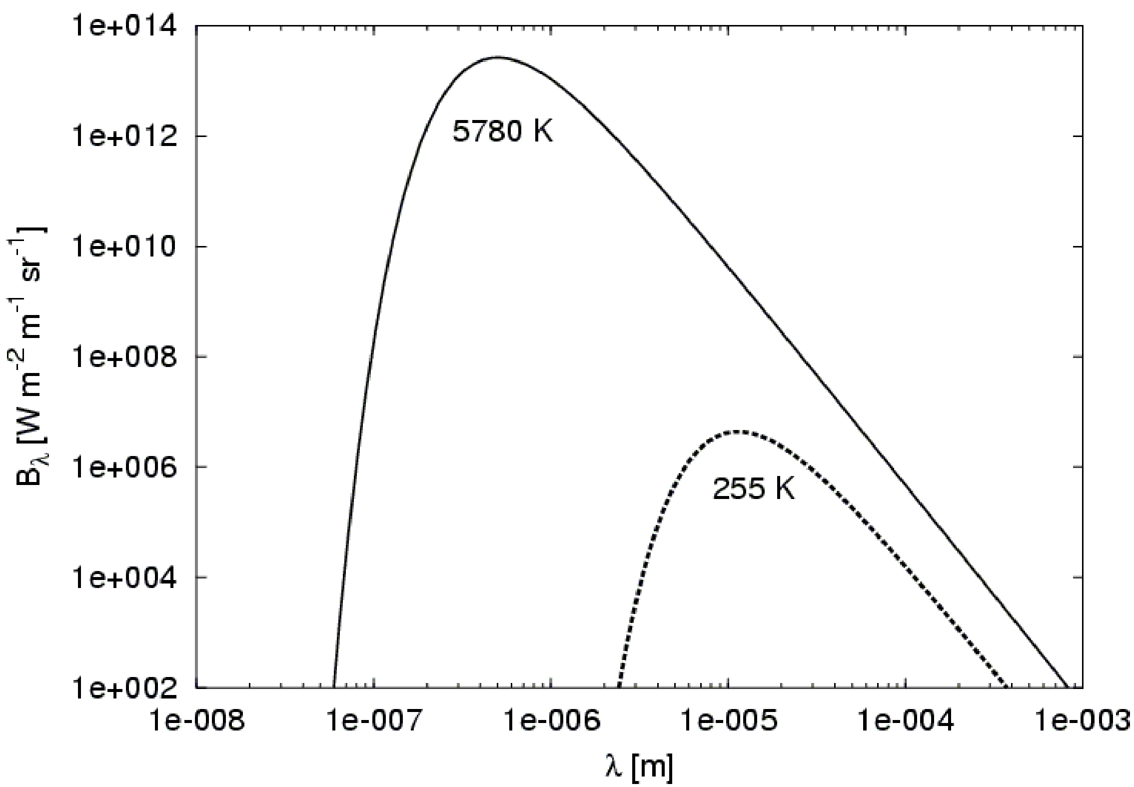
c	=	$2.998 \cdot 10^{8}$ m s^{-1}, speed of light in vacuum
h	=	$6.626 \cdot 10^{-34}$ J s, Planck constant
k	=	$1.381 \cdot 10^{-23}$ J K^{-1}, Boltzmann constant
T	=	absolute temperature [K]

Note that B_λ has units of a spectral radiance. Spectral quantities are related to their total counterparts (p. 1) through:

$$N_\lambda \equiv \frac{dN}{d\lambda}$$

Blackbody radiation (2)

The Planck function is displayed in the following figure for two temperatures, the first (5780 K) roughly corresponding to the temperature of the solar photosphere, the second (255 K) to the effective temperature of the earth-atmosphere system.



Blackbody radiation (3)

The wavelength λ_{max} were the maximum emission takes place is found by setting $dB_\lambda/d\lambda = 0$:

$$\lambda_{max}\, T \;=\; 2.898 \cdot 10^{-3}\ \mathrm{K\,m}$$

This is Wien's displacement law.

The total radiance is found by integrating B_λ over all wavelengths:

$$B(T) \;=\; \int_0^\infty B_\lambda\, d\lambda \;=\; \frac{\sigma T^4}{\pi}$$

where

$\sigma \;=\; 5.67 \cdot 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$, Stefan-Boltzmann constant.

Since the black-body emission is isotropic, the corresponding irradiance is:

$$F(T) \;=\; \frac{\sigma T^4}{\pi} \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta \;=\; \sigma T^4$$

This is Stefan-Boltzmann's law.

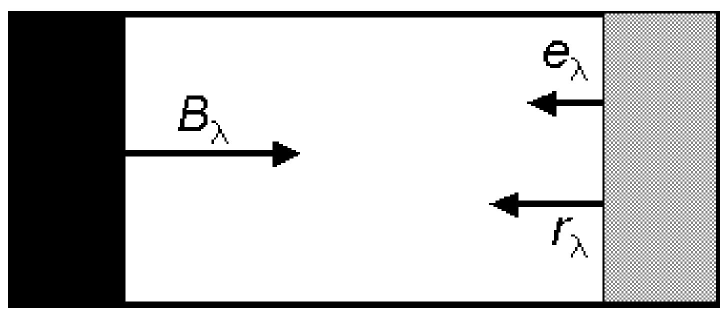
Kirchhoff's law

Contrary to a blackbody, which absorbs all of the incident radiation, a so-called gray body reflects part of the incident radiation (with a reflectivity R_λ). Hence, in thermal equilibrium with a blackbody a gray body can only emit a radiant energy

$$e_\lambda \leq B_\lambda$$

since (see figure)

$$B_\lambda = e_\lambda + R_\lambda$$



Kirchhoff's law (2)

The emissivity ε_λ of a gray body is defined as:

$$\varepsilon_\lambda \;\equiv\; \frac{e_\lambda}{B_\lambda} \;\equiv\; 1 - R_\lambda \;\equiv\; A_\lambda \;\leq\; 1$$

where

R_λ = reflectivity

A_λ = absorptivity

Hence for a gray body $\varepsilon_\lambda = A_\lambda$. This is Kirchhoff's law.

An 'average' emission coefficient ε can be defined by requiring that the total radiant energy emitted by a gray body follows Stefan-Boltzmann's law as:

$$F \;=\; \pi \int_0^\infty e_\lambda \, d\lambda \;=\; \pi \int_0^\infty \varepsilon_\lambda \, B_\lambda \, d\lambda \;=\; \pi \tilde{\varepsilon}_\lambda \int_0^\infty B_\lambda \, d\lambda \;\equiv\; \varepsilon \sigma T^4$$

Emission coefficient

Most natural surfaces have an emission coefficient $\varepsilon \sim 1$; for vegetation, for instance, $\varepsilon \cong 0.9$ (see Oke, 1987 or Garratt, 1992). Snow, too, behaves in the longwave range almost like a blackbody, with $\varepsilon = 0.99$ for fresh snow. Metals, on the other hand, are poor emitters: $\varepsilon \cong 0.03$ for aluminum, $\varepsilon \cong 0.20$ for iron, $\varepsilon = 0.02$ for silver.

The effective emissivity of the atmosphere, ε_a, depends on the concentrations of the greenhouse gases and the presence of clouds.

Note that the total content u of a constituent is usually expressed in terms of the total mass per unit cross-sectional area (in units of [kg m^{-2}] but more typically [g cm^{-2}]), which is the path integral of the density:

$$u \equiv \int \rho \, dz$$

Radiatively active gases in the atmosphere

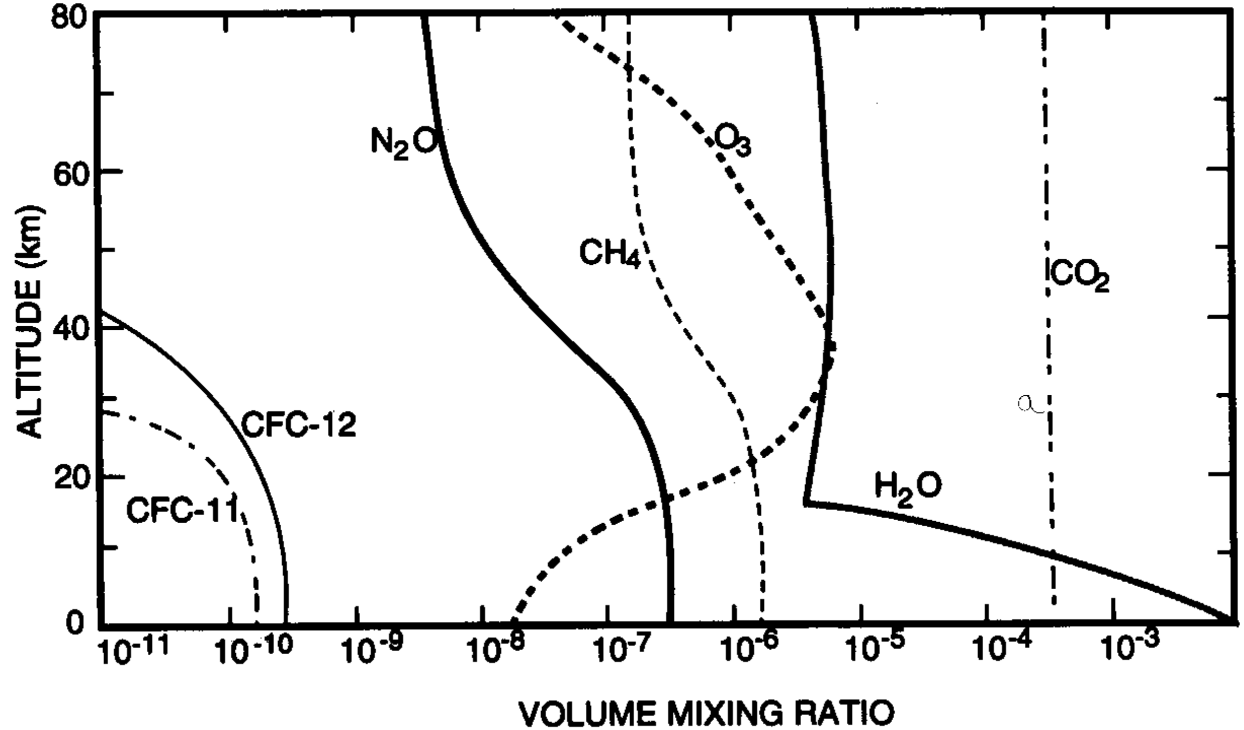
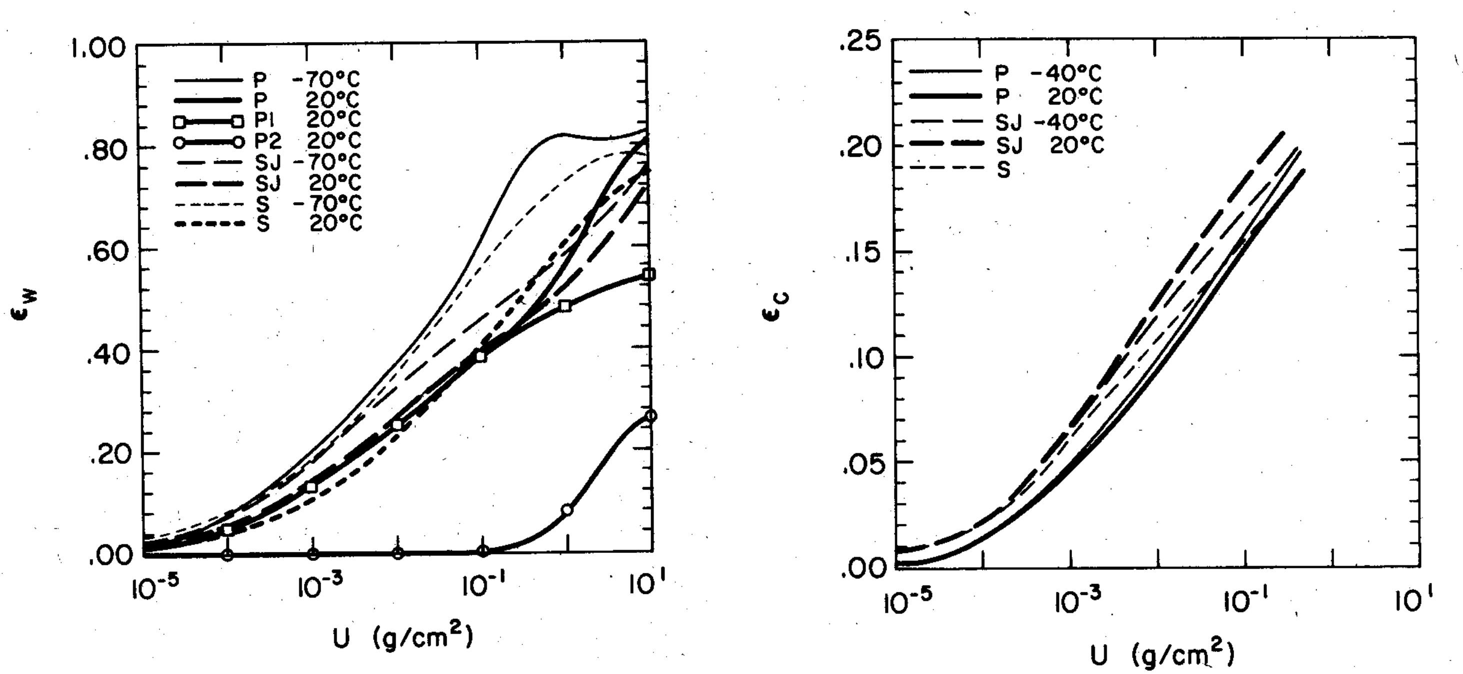


Figure 1.20 Mixing ratios of radiatively active trace species as functions of altitude. *Source:* Goody and Yung (1989).

Water vapor and carbon dioxide

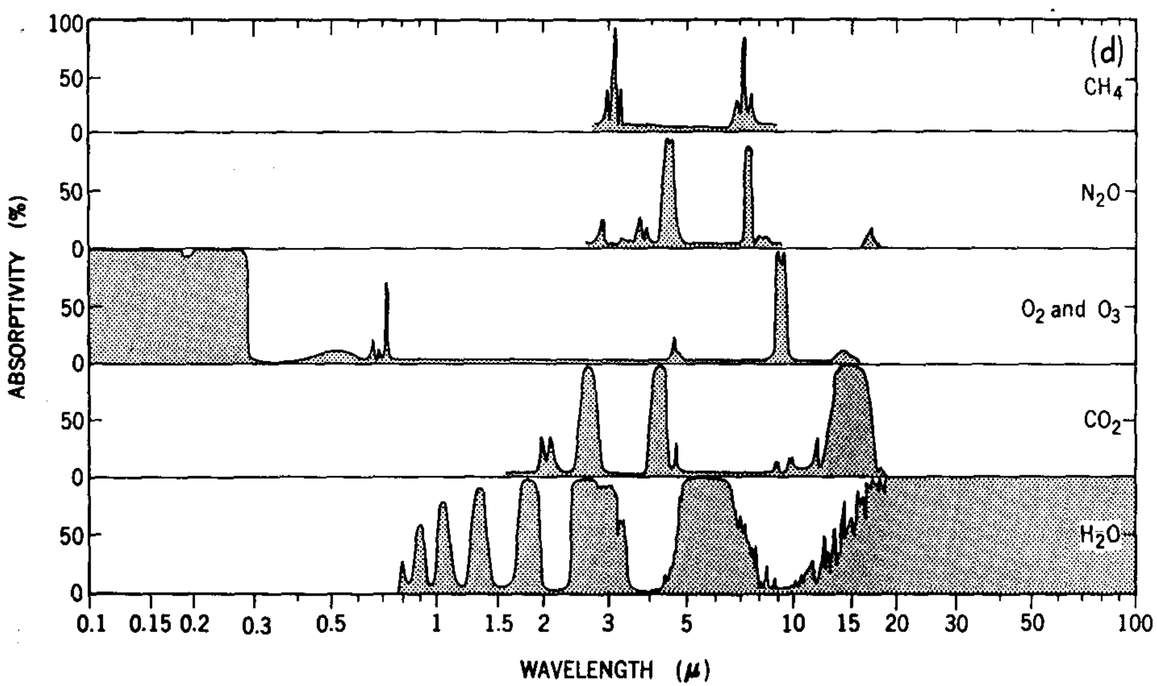
Effective emissivity of water vapor (left) and carbon dioxide (right).



After: Cerni, T. and T. Parish, 1984: A radiative model of the stable nocturnal boundary layer with application to the Polar night. *J. Clim. Appl. Meteor.*, 23, 1563-1572

Absorption spectra

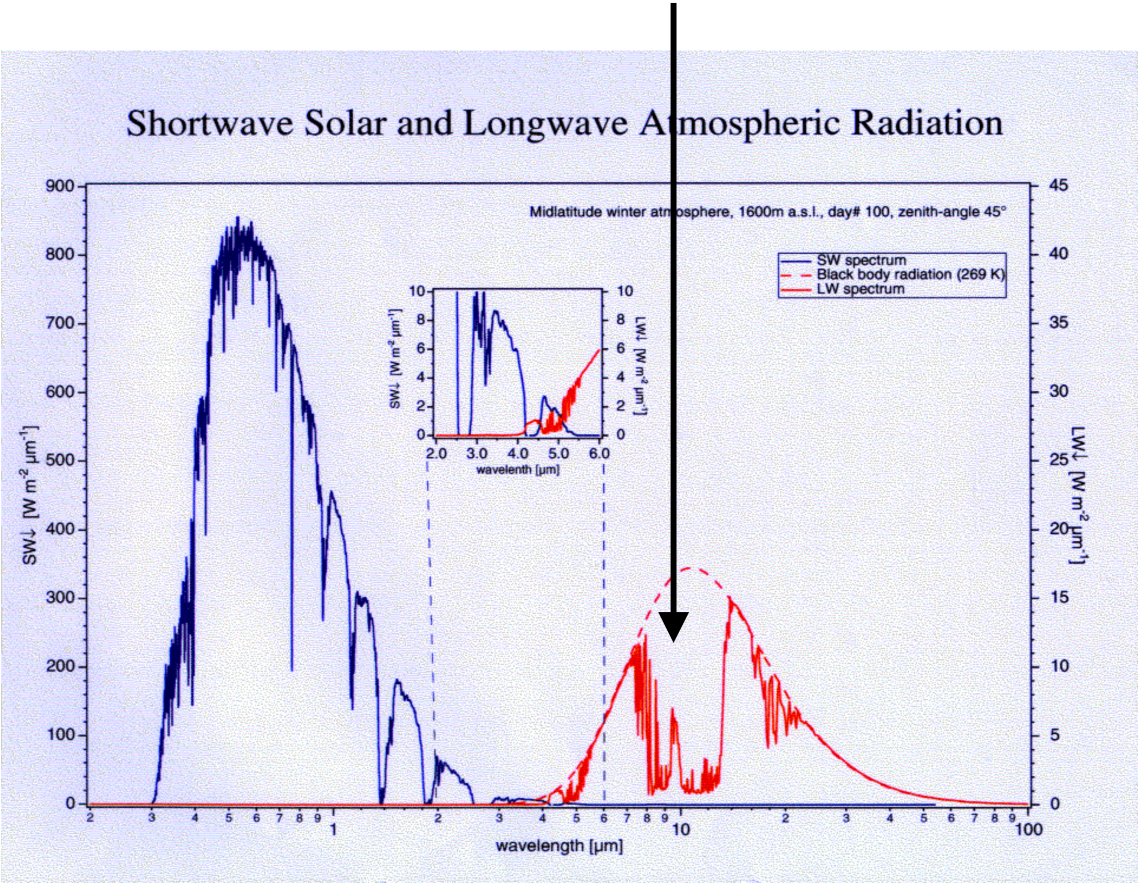
Note, however, that molecular compounds such as H_2O and CO_2 do not emit continuously over the whole spectral range. The implications for the atmospheric absorption (emission) are shown in the following figure.



Absorption spectra for various atmospheric gases between the top of the atmosphere and the surface. From: Peixoto and Oort (1992).

The atmospheric window

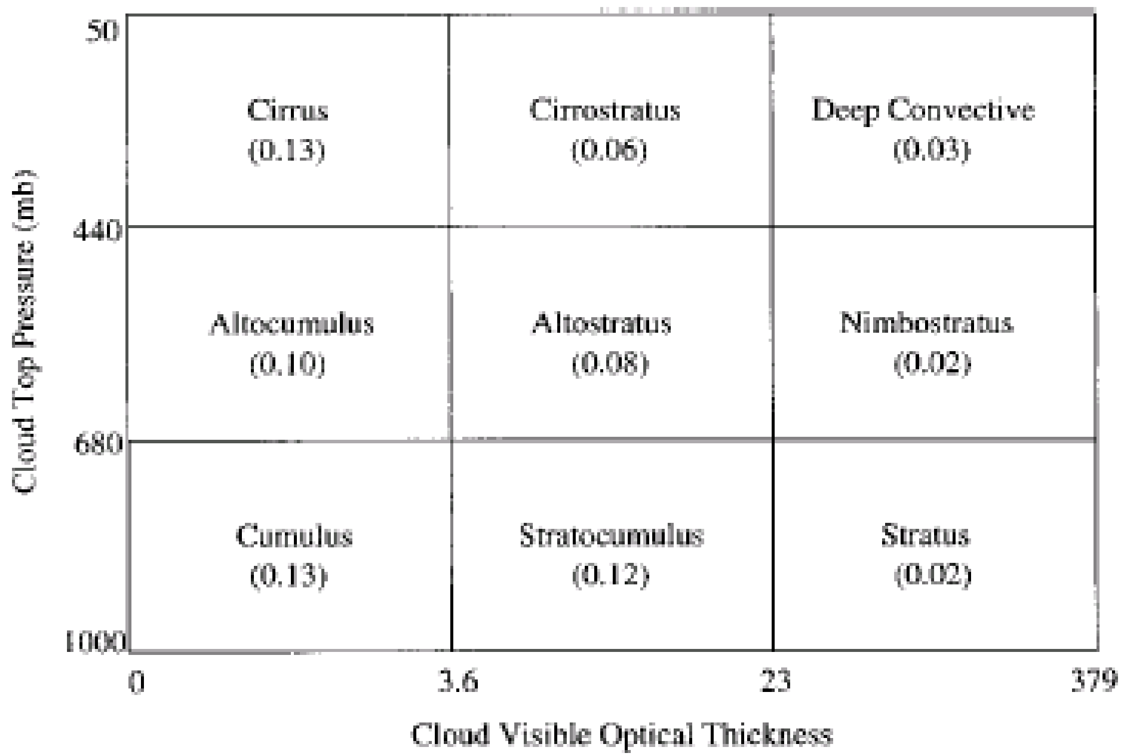
This gives rise to the so-called atmospheric window.



Solar and atmospheric radiation at Davos. Courtesy of Rolf Philippona, MeteoSwiss, Payerne.

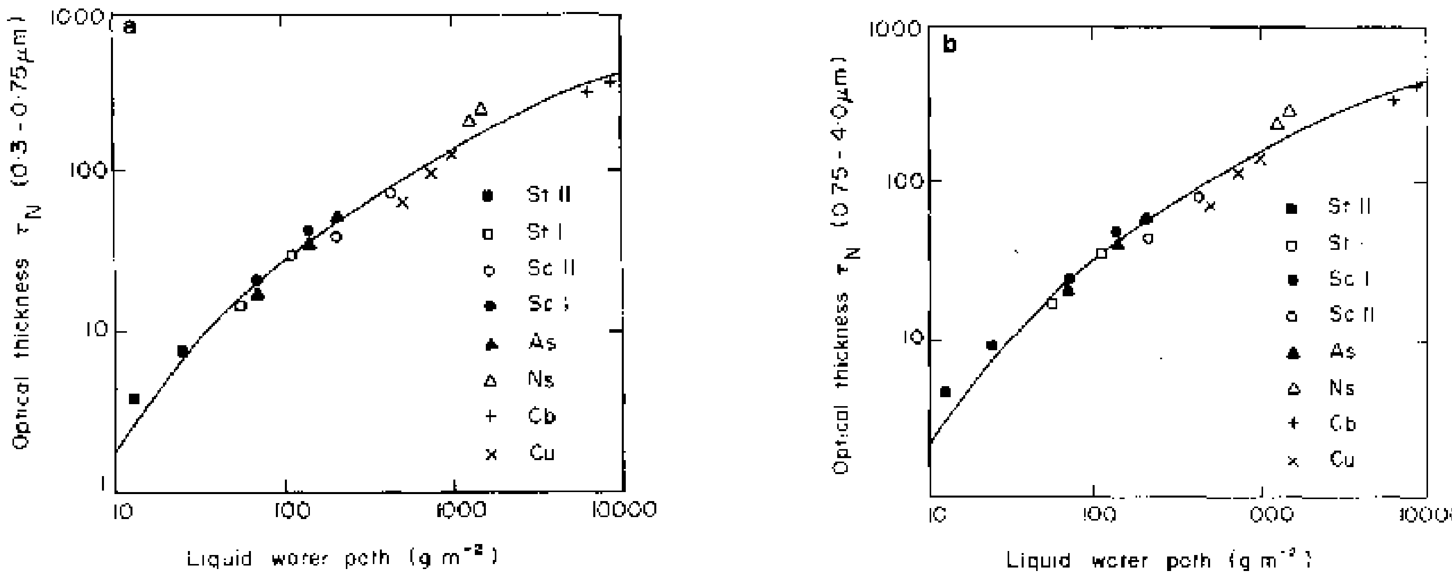
Clouds

A simple classification scheme for clouds



After: Chen, T., W. B. Rossow and Y. Zhang, 2000: Radiative effects of cloud-type variations. *J. Climate*, **13**, 264-286.

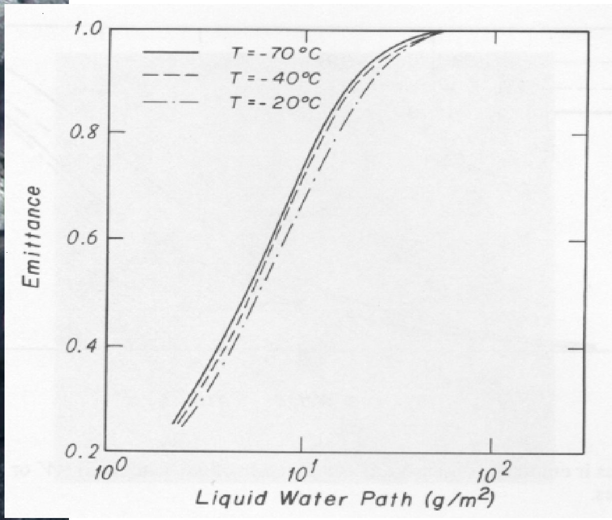
Clouds water content and optical thickness

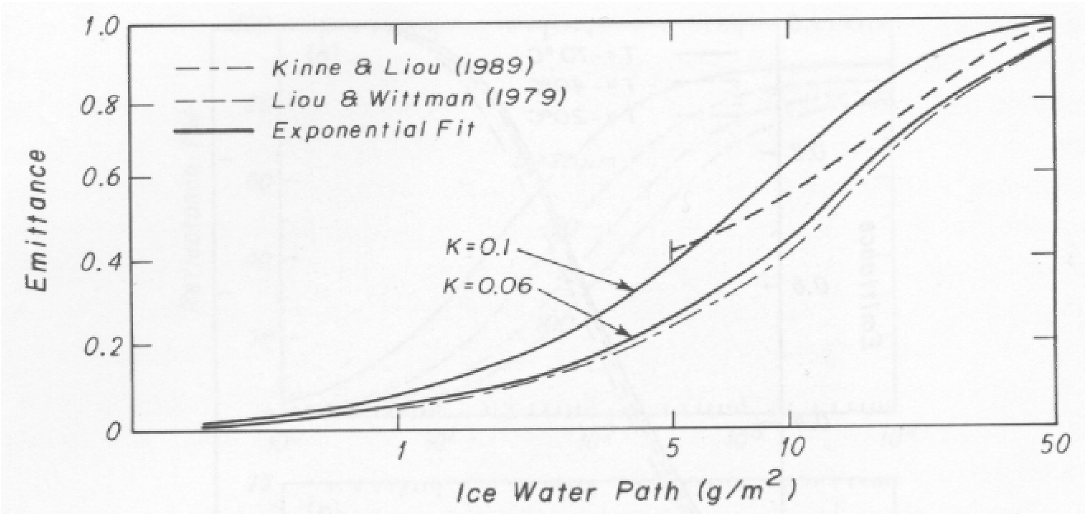


After: Stephens, G.L., 1978: Radiation profiles in extenden water clouds. *J. Atmos. Sciences*, **35**, 2123-2132 .

Clouds emissivity

The effective emissivity of clouds also very much depends on the water (left) or ice content (right).





After: Liou, K.N., 1992: *Radiation and Cloud Processes in the Atmosphere. Theory, Observation, and Modeling*. Oxford University Press, New York, 487 pp.

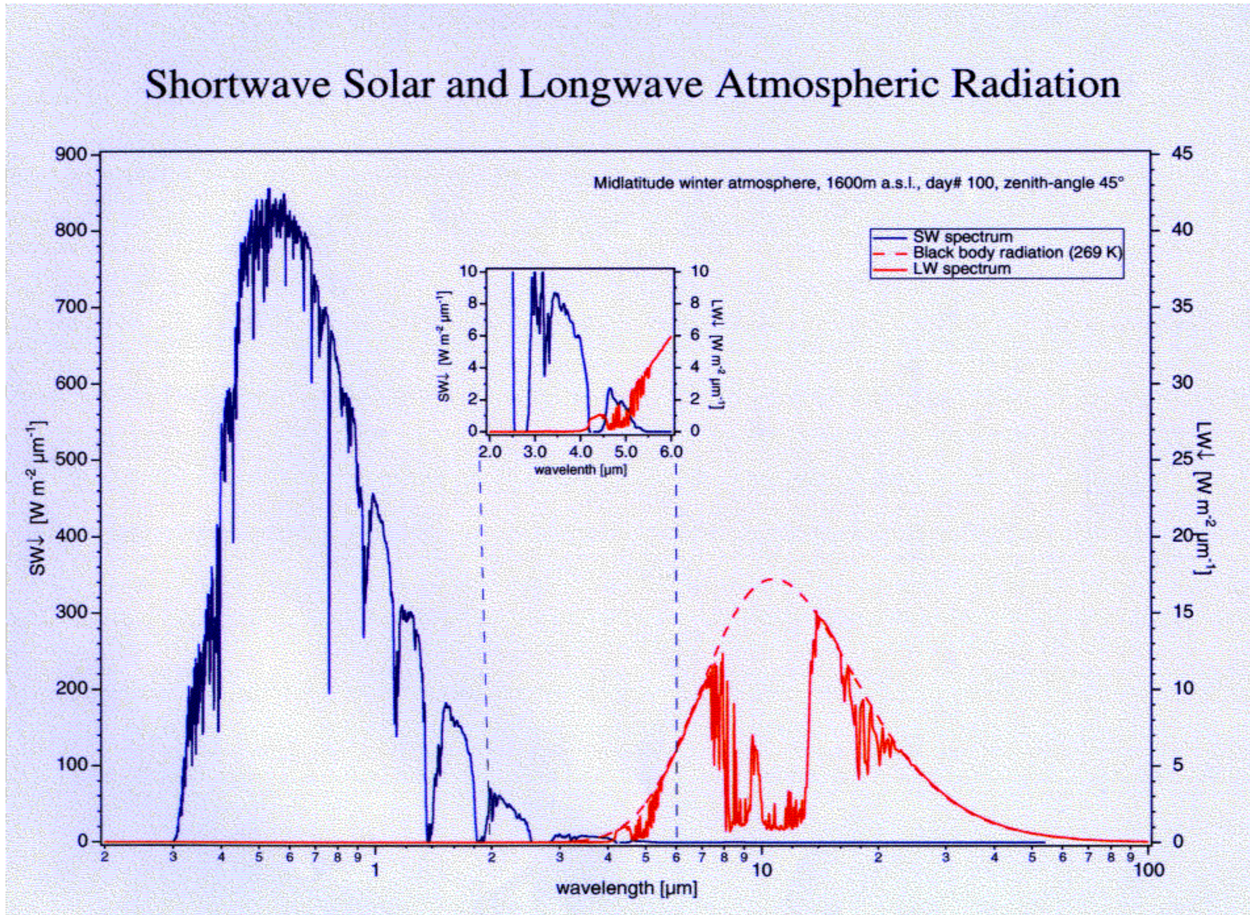
Short- and longwave radiation

Solar radiation as received on the earth's surface mainly stems from the sun's photosphere. This has an effective temperature of 5800 K, more than an order of magnitude larger than the mean temperature at the earth's surface (288 K).

As seen on p. 7, this contrast gives rise to a considerable difference in the Planck function. A comparable difference is found when measuring the spectrum of the incoming solar radiation (direct beam) and of the radiant energy emitted by the atmosphere (see figure on next page).

An overlap is found in the latter case at a wavelength of approximately 4 μm, which is used to discriminate between the so-called shortwave radiation ($\lambda < 4$ μm, basically the solar radiation) and the so-called longwave radiation ($\lambda > 4$ μm, terrestrial and atmospheric radiation).

Short- and longwave radiation (2)

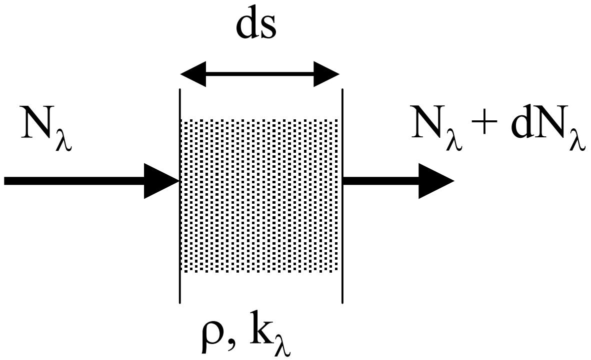


Solar and atmospheric radiation at Davos. Courtesy of Rolf Philippona, MeteoSwiss, Payerne.

Radiative transfer

A pencil of radiation traversing a medium will be weakend by its interaction with matter. This interaction is called extinction or attenuation, an overall designation for the processes of absorption and scattering.

We assume that the medium has a density ρ and is characterized by a mass extinction coefficient of k_λ.



According to the above figure and to first order:

$$dN_\lambda = -\rho\, k_\lambda\, N_\lambda\, ds$$

Radiative transfer (2)

What about the units? dN_λ and N_λ are both in units of W m^{-2} m^{-1} sr^{-1}, whereas ρ and ds are in units of kg m^{-3} and m, respectively. It follows that k_λ must be in units of m^2 kg^{-1}, which can be interpreted as the mass specific cross section.

Can you intuitively explain these units and the meaning of k_λ?

Let us consider the following picture. We assume that density and volume are the same in both cases. In this case the total area of the particles on the right is larger than the one on the left. Why?

density ρ, volume V,
particles radius ~ r,
particles number n

density ρ, volume V,
particles radius ~ r',
particles number n' > n

Radiative transfer (3)

Since density and volume are the same, so must be the mass. Therefore

$$\rho\, n' \frac{4}{3} \pi\, r'^3 = \rho\, n \frac{4}{3} \pi\, r^3$$

or

$$r' = \left(\frac{n}{n'}\right)^{\frac{1}{3}} r < r$$

For the total area:

$$n' 4 \pi r'^2 = n' 4 \pi \left(\frac{n}{n'}\right)^{\frac{2}{3}} r^2 = \left(\frac{n'}{n}\right)^{\frac{1}{3}} n 4 \pi r^2 > n 4 \pi r^2$$

In summary, the extinction coefficient accounts for the effects of the distribution of matter on radiation.

Radiative transfer (4)

On the other hand, the intensity can be strengthened by emission of the material (plus scattering from all other directions into the pencil under consideration). We define a source function coefficient j_λ such that the increase in intensity is given by:

$$dN_\lambda = \rho \, j_\lambda \, ds$$

By combining attenuation and strengthening we have:

$$dN_\lambda = -\rho \, k_\lambda \, N_\lambda \, ds + \rho \, j_\lambda \, ds$$

We further define a source function J_λ as $J_\lambda \equiv j_\lambda / k_\lambda$. It follows that the above equation can be rearranged to give:

$$\frac{dN_\lambda}{\rho \, k_\lambda \, ds} = -N_\lambda + J_\lambda$$

This is the <u>general equation of radiative transfer</u>.

Beer-Bouguer-Lambert law

If scattering and emission can be neglected:

$$\frac{dN_\lambda}{\rho k_\lambda\, ds} = -N_\lambda$$

With $N_\lambda(s = 0) = N_{\lambda 0}$, the equation can be integrated to yield:

$$N_\lambda(s) = N_{\lambda 0} \exp\left(-\int_0^s \rho k_\lambda ds \right)$$

If k_λ is independent of s, then

$$N_\lambda(s) = N_{\lambda 0} \exp\left(-k_\lambda \int_0^s \rho\, ds \right) \equiv N_{\lambda 0} \exp(-k_\lambda u)$$

where the optical path u has been defined as $u = \int_0^s \rho\, ds$.

This is Beer's law, or Beer-Bouguer-Lambert law.

Beer-Bouguer-Lambert law (2)

If k_λ depends on s, then it is more convenient to define the so-called optical depth τ_λ and the transmissivity (spectral transmittance) T_λ as:

$$\tau_\lambda(0,s) \equiv \int_0^s k_\lambda \rho \, ds \qquad \text{and} \qquad T_\lambda \equiv \exp(-\tau_\lambda) \equiv \frac{N_\lambda}{N_{\lambda 0}}$$

For a non-scattering medium, the fraction of radiation absorbed by the medium is:

$$A_\lambda \equiv 1 - T_\lambda$$

where A_λ is the absorptivity (all of the above are monochromatic or spectral quantities). If scattering takes place, a certain portion of the incident radiation can be reflected back into the incident direction. The ratio of the reflected (backscattered) to the incident intensity is called monochromatic reflectivity, R_λ. In this case:

$$T_\lambda + A_\lambda + R_\lambda = 1$$

Beer-Bouguer-Lambert law (3)

Consider for instance the properties of water clouds as discussed by Stephens (1978).

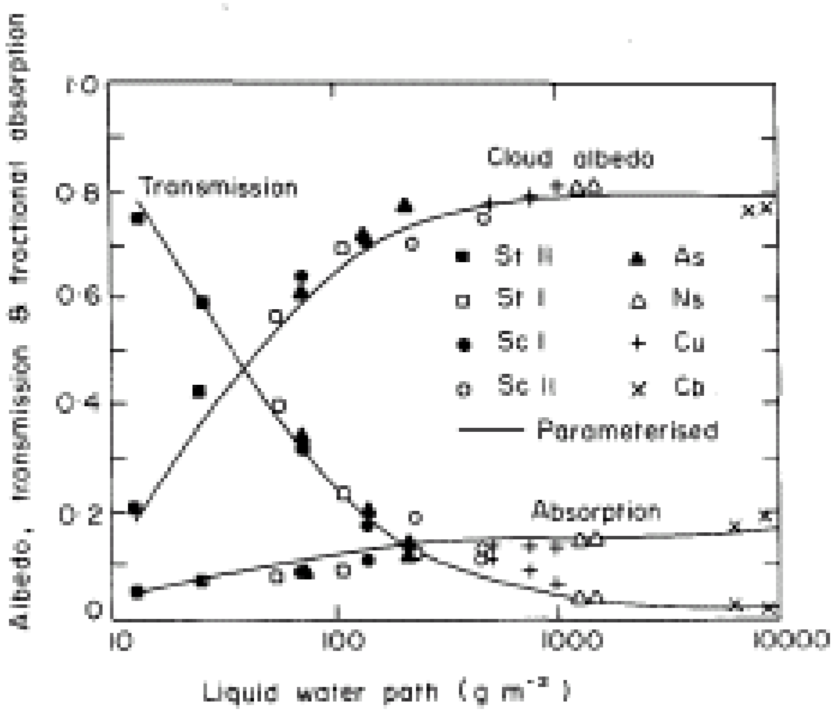


FIG. 3. Comparisons between the shortwave absorption, transmission and cloud albedo as determined by the theoretical model for the given cloud types (illustrated points) and the parameterized scheme (solid lines). The cosines of the solar zenith angle (μ_0) is 0.8 and a zero surface albedo (a_s) is assumed.

After: Stephens, G.L., 1978: Radiation profiles in extenden water clouds. *J. Atmos. Sciences*, **35**, 2123-2132 .

Schwarzschild’s equation

Consider a non-scattering medium which is a blackbody and which is in local thermodynamic equilibrium. A beam of radiation passing through it will undergo absorption; at the same time emission takes place. In this case the source function is given by the Planck function, $J_\lambda \equiv B_\lambda(T)$. Then the equation of transfer may be written as:

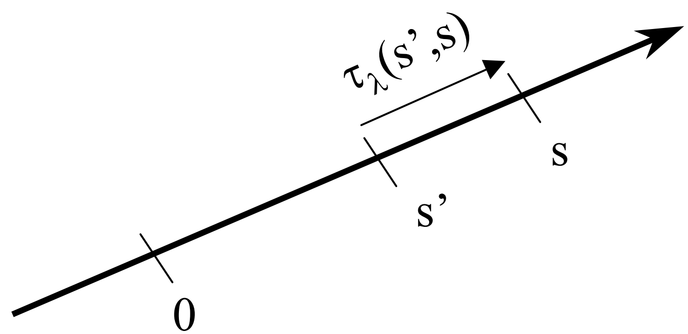
$$\frac{dN_\lambda}{d\tau_\lambda} = -N_\lambda + B_\lambda(T)$$

which is called the <u>Schwarzschild equation</u>. Its formal solution is:

$$N_\lambda(s) = N_\lambda(0)\,e^{-\tau_\lambda(0,s)} + \int_0^s B_\lambda\left[T(s')\right]e^{-\tau_\lambda(s',s)}k_\lambda\,\rho\,ds'$$

where

$$\tau_\lambda(s',s) = \int_{s'}^{s} k_\lambda\,\rho\,ds$$



Schwarzschild's equation (2)

Only rarely can Schwarzschild's equation be solved analytically. Most radiation transfer codes, needed in remote sensing studies, are therefore designed to solve it numerically.

Scattering

Scattering is the physical process by which a particle (or molecule) in the path of an electromagnetic wave continuously extracts energy from the incident wave and reradiates that energy in all directions. In the atmosphere, the particles responsible for scattering range from gas molecules (~ 10^{-4} μm) to large raindrops and hail particles (~ 10^{4} μm). We can broadly distinguish the following categories:

- solid aerosols (0.1 to 1 μm), irregular shape, variable refractive index;
- haze water drops (0.1 to 1 μm), spherical, known refractive index;
- cloud water drops (1 to 10 μm), spherical, known refractive index;
- cloud ice particles (1 to 100 μm), irregular shape, known refractive index.

Based on the size of the scattering particles, we distinguish between

- <u>Rayleigh scattering</u>, particle diameter << wavelength of the incident beam
- <u>Mie scattering</u>, particles diameter ~ wavelength of the incident beam

Mie theory provide a framework for describing scattering caused by spherical particles.

Scattering (2)

Rayleigh scattering is characterized by symmetry between forward and backward scattering. As the particles become larger, an increasing proportion of the incident radiation is scattered in the forward direction. Sketches of the angular pattern of the scattered intensity from particles of various sizes are shown here.

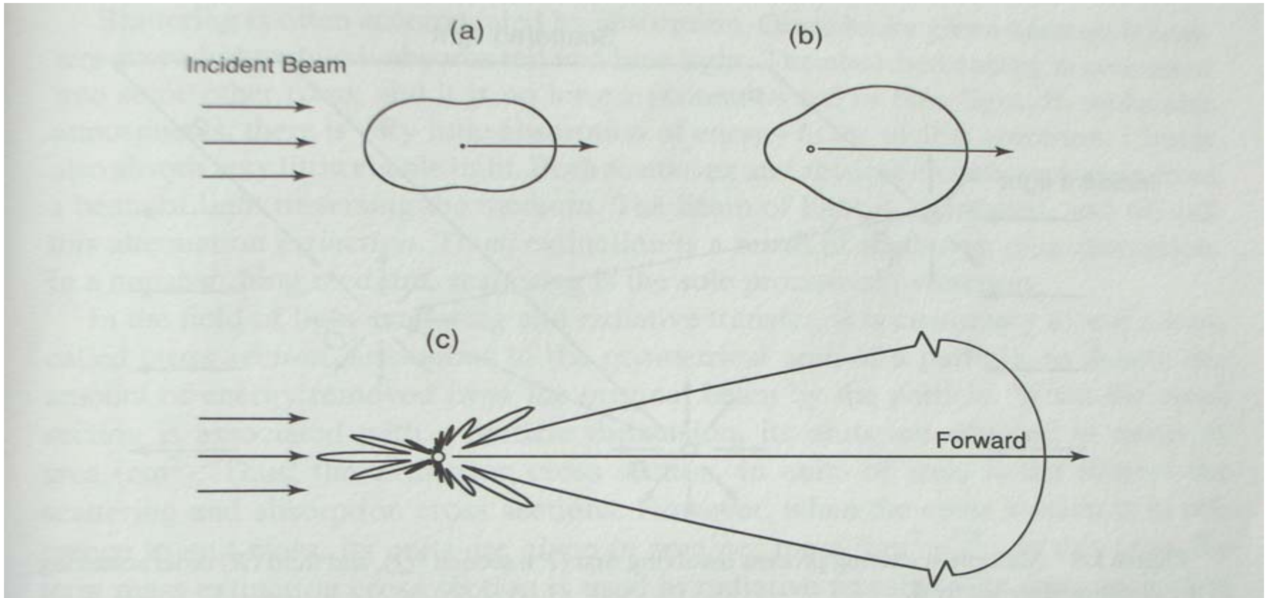


Figure 1.4 Demonstrative angular patterns of the scattered intensity from spherical aerosols of three sizes illuminated by the visible light of 0.5 μm: (a) 10^{-4} μm, (b) 0.1 μm, and (c) 1 μm. The forward scattering pattern for the 1 μm aerosol is extremely large and is scaled for presentation purposes.

After: Liou, K.N., 2002: An Introduction to Atmospheric Radiation (2nd Ed.). Academic Press, Amsterdam

Scattering (3)

Rayleigh scattering decreases with increasing wavelength of the incident beam according to:

$$\frac{N_{sc}}{N_0} \propto \lambda^{-4}$$

where N_0 and N_{sc} are the total intensities of the incident and scattered radiation. This relation explains why the sky appear blue under cloudless conditions.

Rayleigh scattering is also responsible for the albedo of the clear-sky atmosphere. It can be shown (see course ‘Theoretical Climatology’) that the planetary albedo of a purely Rayleigh atmosphere is ~ 0.2 (assuming a surface albedo of 0.16), which is somewhat higher than the observed clear-sky planetary albedo of 0.17.

The dependence of Mie scatter on the wavelength of the incident beam is more complex. As a rule of thumb:

$$\frac{N_{sc}}{N_0} \sim \lambda^{-1.3}$$

Diffuse reflection

Diffuse reflection may occur in the presence of large particles (cloud water drops). This type of reflection is independent of the wavelength of the incident beam and is responsible for the white color of clouds.