1. Gas properties

Starting from the ideal gas law:

$$pV = Nk_bT$$

where p is the pressure, V the volume and T the temperature of the gas, $k_b = 1,3806505 \times 10^{-23}$ J K^{-1} the Boltzmann constant and N being the number of particles in the gas-volume. Derive the following form of the ideal gas equation:

$$pV = nRT$$

using $N_A = 6,0221415 \times 10^{23}$ mol^{-1} (Avogadro constant) and calculating the universal gas constant R with units J mol^{-1} K^{-1}.

Finally derive the gas law for dry air:

$$p_d = \rho_d R_d T$$

where $\rho_d \left(= \frac{m_d}{V_d}\right)$ is the density of dry air and R_d is the specific gas constant for dry air. In reality the atmosphere is never in pure dry condition (compare Fig. 1). Calculate R_d and also the specific gas constant for water vapour R_v using the molecular weights of oxigen (M(O)=16 g mol^{-1}) nitrogen (M(N)=14 g mol^{-1}) and hydrogen (M(H)=1 g mol^{-1}) knowing that the atmosphere consists of about 80% N_2 and 20% O_2. (Hint: Water = H_2O)

(a) Explain the definition of the virtual Temperature (T_v) in one sentence.

(b) Show that the ideal gas law $p = \rho R_d T$ is a good approximation for the atmosphere, i.e. can be used instead of $p = \rho R_d T_v$ (compare T_v and T). Quantify your result by choosing specific humidity q from three regions in Fig. 1.

(c) Explain the following statement: Dry air is more dense than moist air.

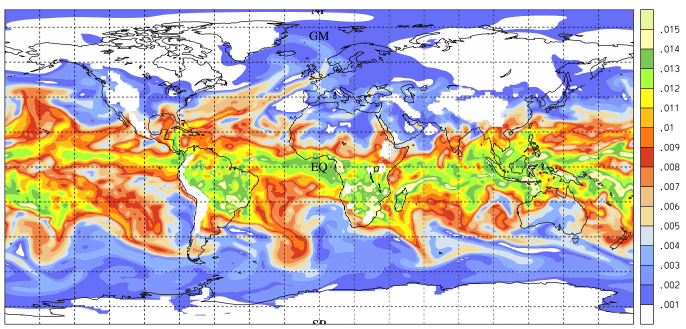

Figure 1: Distribution of the specific humidity [g/kg] at 850 hPa for Dec 1 2000 00 UTC, using the ERA40 dataset.

2. Air properties

Assume a vertical column of air is

- isothermal (i.e. at uniform temperature $T = 273$ K)
- hydrostatic equilibrium (with $g = 9,81$ m s^2)
- and behaves like an ideal dry gas ($\mathrm{R}_d = 287$ J kg^{-1} K^{-1})

Show that the variation of pressure with height satisfies the relationship

$$p(z) = p_0\, e^{-\frac{g}{RT}z}$$

Also estimate:

(a) At what height will the pressure have decreased to $(1/e)$ of its value at the surface?

(b) What is the typical pressure difference between Jungfraujoch (3454 m) and Interlaken (560 m)?

(c) The change in the value of the answer to (b) if the mean temperature in the layer changes by $+15\,\mathrm{K}$.

3. Viscosity

Viscosity is a measure of the resistance of a fluid to deformation under shear stress. Water and most gases, are known as Newtonian fluids, which means that the shear stress is proportional to the strain rate.

(a) Write down and explain the definition of the dynamic and kinematic viscosity.

(b) By increasing the temperature, the viscosity of air also increases but the viscosity of water decreases (compare Fig. 2.6 in lecture notes). Give a short explanation.

4. Dynamics

Material/total derivative of wind velocity $\mathbf{v}$:

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}$$

(a) Name each of the three terms in the equation.

(b) Under which conditions is the advective rate of change equal to the total rate of change?

(c) What is the technical term for this flow characteristic?

(d) What happens to streamlines and partical paths/trajectories under these conditions compared to other conditions? Explain!

(e) Write down $\mathbf{v} \cdot \nabla$ and $\nabla \cdot \mathbf{v}$ in cartesian form und show that they are significantly different. Which of them is a scalar function and which is a differential operator?