Problem sheet II

1. Potential Temperature and lapse rate

The potential temperature is given by

$$\theta = T \left(\frac{p_0}{p} \right)^{\kappa} \tag{1}$$

where T is the actual Temperature at pressure level p, p_0 is a reference pressure and $\kappa = \frac{R}{c_p}$.

(a) Under which condition is the potential temperature conserved ($\frac{d\theta}{dt} = 0$)? (Just state the technical term for this thermodynamic process.)

(b) Name some physical processes in the atmosphere which usually prohibit the use of the assumption $\frac{d\theta}{dt} = 0$.

(c) Calculate the lapse rate of the temperature ($\frac{dT}{dz}$) in a neutrally stratified atmosphere ($\frac{d\theta}{dz} = 0$). (Hint: You will need to use the hydrostatic and ideal gas equation.)

2. Coriolis force

You go to the 'Teufelsrad' at Octoberfest in Munich to do some experiments for your lecture Environmental Fluid Dynamics. The Teufelsrad is basically a horizontal turn table rotating clockwise about a vertical axis where people have to try to stay on it as long as possible.

Assumptions:

- You wear shoes with perfect grib (meaning you are fixed on the turn table).
- You never get dizzy.

(a) Standing on the edge of the wheel you are throwing a ball towards the center of the turn-table. Will it reach the center? If not, to which direction will it deviate relative to the line between you and the center of the Teufelsrad.

(b) Now you are standing at the center of the turn-table throwing the ball outwards. To which direction of you will the ball deviate?

More Coriolis force:

- As the polar tourism increases some railway companies are starting to think about circum-arctic and circum-antarctic railway tracks. Which direction (east or west) should the trains go in each hemisphere, respectively, to have the Coriolis force not acting too hard against the tracks when the train is going at hight speeds.

Don't be fooled by these questions. There is really some thinking envolved. Each statement you make has to be explained with a few words.

3. Gravity versus rotation

A geo-stationary satellite is orbiting above the equator. The mass of the earth is $M = 5.988 \times 10^{24}$ kg, the gravitational constant is $G = 6.673 \times 10^{-11}$ N m^2 kg^{-1} and the radius of the earth is $R = 6.37 \times 10^6$ m. (Hint: You will need to use eq. (2.1) from the script.)

(a) Which forces must be in balance?

(b) Calculate the height of the satellite above the earths surface.

The equations of motion in a cylindrical polar coordinate system ($\mathbf{x} = (r, \varphi, z)$) are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \varphi} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \quad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \varphi} + w\frac{\partial v}{\partial z} + \frac{vu}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \varphi} \quad (3)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \varphi} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad (4)$$

Assumptions:

- Steady state of the fluid motion.
- Fluid is in hydrostatic balance with constant density $\rho = const.$
- Fluid is rotating with velocity $\mathbf{v} = (0, \Omega r, 0)$ in a Cylinder with radius R.
- The height of the fluid at rest before 'spinning up' the Cylinder is H.

(a) Rewrite the equations of motion (2-4) considering the assumptions above.

(b) Calculate the surface height z as a function of the radius r, the starting height H of the fluid at rest, the radius of the cylinder R and Ω.

(c) Calculate the surface pressure distribution p_0 as a function of the radius r, the starting height H, the radius of the cylinder R and Ω.