1. Gas properties

(a) The virtual temperature is a fictitious temperature that takes into account moisture in the air. The formal definition of virtual temperature is the temperature that dry air would have if its pressure and specific volume were equal to those of a given sample of moist air.

(b) Virtual temperature:

$$T_v = T(1 + 0.6078\, q)$$

Typical specific humidity in the pole regions:

$$q = 2\, g/kg \rightarrow T_v = 1.001 \cdot T$$

Typical specific humidity in the extratropics:

$$q = 7\, g/kg \rightarrow T_v = 1.004 \cdot T$$

Possible specific humidty in the tropics:

$$q = 16\, g/kg \rightarrow T_v = 1.009 \cdot T$$

Therefore $T_v \sim T$

(c) The amount of water vapor in the air effects the density. Water vapor is a relatively light gas when compared to diatomic Oxygen and diatomic Nitrogen - the dominant components in air. When water vapor content increases, the amount of Oxygen and Nitrogen decreases per unit volume and the density will decrease because the mass is decreasing. Dry air is more dense that humid air!

2. Water properties

$$\rho = \rho_* \left(1 - \alpha_* \Theta_*^2\right)$$

Assume: $T_* = 4\mathrm{K}$, $\alpha_* = 6.8 \cdot 10^{-6}\mathrm{K}^{-2}$, $\Theta_* {=} T - T_* {=} 10\,\mathrm{K}$
Searched is the relative change of density:

$$\frac{\Delta\rho}{\rho} = \frac{\rho\,(\Theta_* + \Delta\Theta_*) - \rho\,(\Theta_*)}{\rho\,(\Theta_* + \Delta\Theta_*)}$$

First possibility: Equation (1) with $\Theta_* \rightarrow \Theta_* {+} \Delta\Theta_*$

$$\begin{aligned}
\Delta\rho &= \rho_* \left(1 - \alpha_* \left(\Theta_* + \Delta\Theta_*\right)^2\right) - \rho_* \left(1 - \alpha\Theta_*^2\right) \\
&= \rho_* \left(1 - \alpha_* \left(\Theta_*^2 + 2\Delta\Theta_*\Theta_* + \Delta\Theta_*^2\right)\right) - \rho_* \left(1 - \alpha_* n\Theta_*^2\right) \\
&= -\alpha_* \rho_* \Delta\Theta_* \left(2\Theta_* + \Delta\Theta_*\right)
\end{aligned}$$

Second possibility: Taylor series of (1)

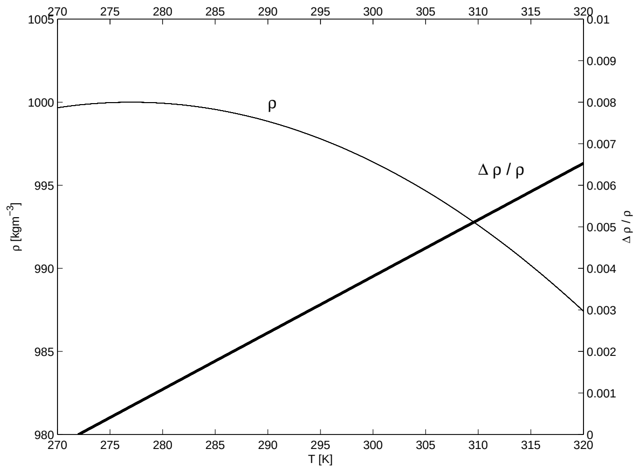
$$\rho\left(\Theta_*+\Delta\Theta_*\right)=\rho\Theta_*+\left.\frac{\partial\rho}{\partial\Theta_*}\right|_{\Theta_*}\Delta\Theta_*+\frac{1}{2}\left.\frac{\partial^2\rho}{\partial\Theta_*^2}\right|_{\Theta_*}\Delta\Theta_*^2+...$$

The last two terms can be written as:

$$\begin{aligned}\Delta\rho &= -\rho_* 2\Theta_*\alpha_*\Delta\Theta_* - \frac{1}{2}\rho_*\alpha_* 2\Delta\Theta_*^2 \\ &= -\rho_*\alpha_*\Delta\Theta_*\left(2\Theta_*+\Delta\Theta_*\right)\end{aligned}$$

$$\begin{aligned}\Rightarrow \frac{\Delta\rho}{\rho} &= \frac{-\rho_*\alpha_*\Delta\Theta_*\left(2\Theta_*+\Delta\Theta_*\right)}{\rho_*\left(1-\alpha_*\left(\Theta_*+\Delta\Theta_*\right)^2\right)} \\ &\cong -\alpha_*\Delta\Theta_*\left(2\Theta_*+\Delta\Theta_*\right) \\ &\cong -6.8\cdot 10^{-6}\mathrm{K}^{-2}\cdot 10\mathrm{K}\cdot\left(2\cdot 10\mathrm{K}+10\mathrm{K}\right) \\ &\cong 2\cdot 10^{-3} \\ &\cong 0.2\%\end{aligned}$$

The water density does not significantly change if the temperature change is only 10 K.



Assumptions:

- isothermal: $T = \text{const.}$ (1)
- hydrostatic: $\frac{\partial p}{\partial z} = -\rho(z) \cdot g$ (2)
- ideal gas equation: $p = \rho \cdot R \cdot T$ (3)

Put (3) into (2):

$$\frac{\partial p}{\partial z} = -\frac{\rho g}{RT} \Longleftrightarrow \frac{\partial p}{\rho} = -\frac{g \partial z}{RT}$$

Integrate from 0 to z:

$$\begin{aligned}
\int_0^z \frac{dp}{\rho} &= \int_0^z -\frac{g}{RT} dz \\
\ln p|_0^z &= -\frac{gz}{RT} + C \\
p(z) &= p_0 \cdot e^{-\frac{gz}{RT}}
\end{aligned}$$

(a) The e-term in the pressure equation shows the decay of the surface pressure with height. Therefore:

$$\frac{gH}{RT} = 1 \Longrightarrow H = \frac{RT}{g}$$

Assumptions: $T = 273\mathrm{K}$, $g = 10\mathrm{ms}^{-2}$, $R = 287\mathrm{Jkg}^{-1}\mathrm{K}^{-1}$(dry air!)

H=7800 m

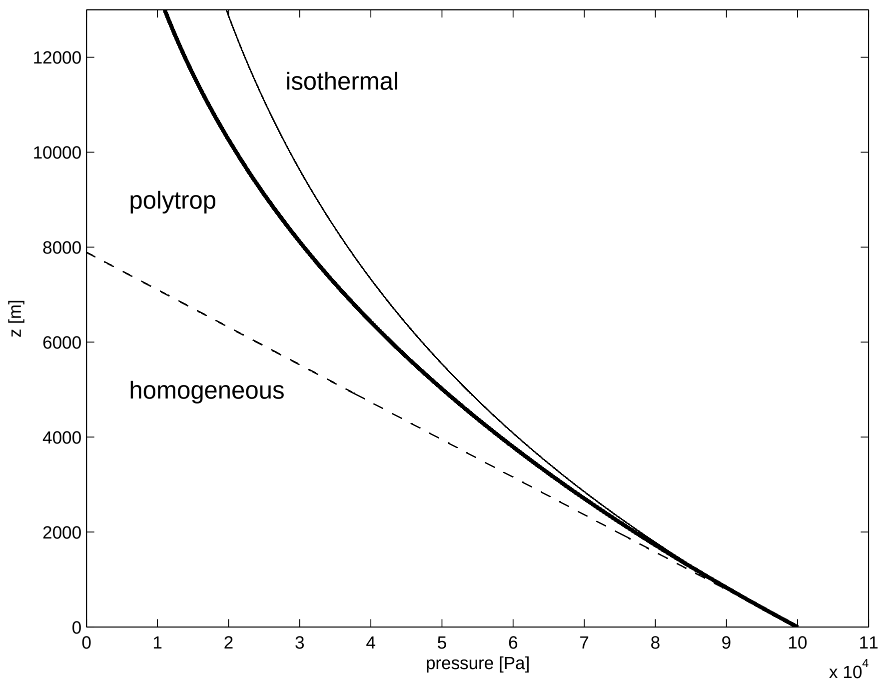
(b) Jungfraujoch: 3454 m, Interlaken: 560 m
Height difference: $\Delta H \cong 2900$ m
T=273 K

$$\begin{aligned}
p_{jj} - p_{int} &= p_0 \cdot e^{-\frac{g}{RT} z_{int}} \left(1 - e^{-\frac{g}{RT}(z_{jj} - z_{int})}\right) \\
p_{jj} &= p_{int} \left(1 - e^{-\frac{g}{RT} \Delta H}\right) \\
&\cong p_{int} \left(1 - e^{-\frac{10\mathrm{ms}^{-2} \cdot 2900\mathrm{m}}{287\mathrm{JK}^{-1}\mathrm{kg}^{-1} \cdot 273\mathrm{K}}}\right) \\
&\cong \mathbf{0.691 \cdot p_{int}}
\end{aligned}$$

(c) $\Delta T = 15\mathrm{K}$, the new Temperature is $T_* = 288\mathrm{K}$

$$\mathbf{p_{jj} \cong 0.704 \cdot p_{int}}$$

The pressure is not very depending on temperature changes!



4. Viscosity

(a) Dynamic viscosity μ: The ratio of shear stress τ to the associated strain rate E':

$$\mu = \frac{\tau}{E'}$$

Kinematic viscosity ν: The ratio of the absolute viscosity μ to the density ρ:

$$\nu = \frac{\mu}{\rho}$$

(b) Air / Gases:
Viscosity in gases arises principally from the molecular diffusion that transports momentum between layers of flow. The kinetic theory of gases allows accurate prediction of the behaviour of gaseous viscosity. In particular that, within the regime where the theory is applicable:
Viscosity is independent of pressure, Viscosity increases as temperature increases.

Water / Liquids:
Our everyday experience suggests that viscosity of a liquid decreases as the temperature increases. Honey becomes "thinner" as it heats and engine oil thickens in very cold temperatures.
In liquids, the additional forces between molecules become important. This leads to an additional contribution to the shear stress. Thus, in liquids:
Viscosity is independent of pressure (except at very high pressure), Viscosity tends to fall as temperature increases.

The dynamic viscosities of liquids are typically several orders of magnitude higher than dynamic viscosities of gases.