Problem sheet IV

Fluid Dynamics

1. Laminar Flow in a water channel

- A sluice gate controls the discharge of water down a channel. If the discharge is increased by 20%, what will be the percentage change in the depth of the water? Compare lecture notes on page 53.
- Is the percentage in the depth dependent upon the viscosity of the water? Is it dependent upon the temperature?

2. Water Flow

If the velocity w in the flow system considered on page 54 in the lecture notes is $10^{-2}\,\mathrm{ms}^{-1}$, how deep is the boundary layer of water at normale temperature? How deep would it be for air with the same specifications?

3. 2D flow field

A two dimensional flow field occupying the domain y>0 is specified in terms of the streamfunction ψ, such that

$$\psi = A \cdot \sin(kx) \cdot e^{-ly}$$

(a) Sketch the streamfunction ψ and the corresponding streamlines.

(b) Derive expressions for the horizontal velocity components (u, v)

(c) Derive expressions for the vertical component of the vorticity ζ

(d) Under what relative values 'k' and 'l' will the streamfunction ψ be a solution of:
$\frac{D}{Dt}\zeta = \nu \nabla^2 \zeta$ where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the vorticity.

Hint: Use a software package such as Maple, Mathematica, Matlab, ... if you have access.

4. Shallow water system

Consider the one-spare dimension shallow water system:

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv &= -g\frac{\partial h}{\partial x} \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu &= 0 \\
\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} &= 0
\end{aligned}$$

or written as:

$$\begin{aligned}
u_z + uu_x - fv &= -gh_x \\
v_z + uv_x + fv &= 0 \\
h_z + vh_x + hu_x &= 0
\end{aligned}$$

Non-dimensionalise this set to the form:

$$
\begin{aligned}
R_0[\tilde{u}_{\tilde{z}} + \tilde{u}\tilde{u}_{\tilde{x}}] - \tilde{v} &= (\exists R_0)\tilde{h}_{\tilde{x}} \\
R_0[\tilde{v}_{\tilde{z}} + \tilde{v}\tilde{v}_{\tilde{x}}] + \tilde{u} &= 0 \\
\tilde{h}_{\tilde{z}} + \tilde{u}\tilde{h}_{\tilde{x}} + \tilde{h}\tilde{u}_{\tilde{x}} &= 0
\end{aligned}
$$

where

$$
\tilde{u} = \frac{u}{U} \qquad \tilde{v} = \frac{v}{V} \qquad \tilde{x} = \frac{x}{L} \qquad \tilde{y} = \frac{y}{L} \qquad \tilde{h} = \frac{h}{H} \qquad \tilde{t} = \left(\frac{U}{L}\right) t \qquad R_0 \frac{U}{fL}
$$

(a) What is the dimensionless parameter $\exists$?

(b) Consider some observed motions such that
$U \sim 1\text{cms}^{-1}$, $L \sim 10\text{km}$ and $H \sim 10\text{m}$
What are the characteristic values of R_0 and $\exists$?

(c) To build a laboratory analogue of the lake flow, what should be the depth H of the model, and how quickly must the laboratory model be rotated? Assume model is such that $U \sim 1/10\text{cms}^{-1}$ and $L \sim 1\text{m}$.