Problem sheet I

1. Gas properties

A good approximation for the density of dry air is given by the ideal gas law:

$$p_d = \rho_d R_d T$$

In reality the atmosphere is never in pure dry condition (compare Fig. 1) and the range of the ideal gas constant shifts between $278\,\mathrm{JK^{-1}kg^{-1}}$ for pure dry air and $461\,\mathrm{JK^{-1}kg^{-1}}$ for water vapour.

(a) Explain the definition of the virtual Temperature (T_v) in one sentence.

(b) Show that the ideal gas law $p = \rho R_d T$ is a good approximation for the atmosphere, i.e. can be used instead of $p = \rho R_d T_v$ (compare T_v and T). Quantify your result by choosing specific humidity q from three regions in Fig. 1.

(c) Explain the following statement: Dry air is more dense than moist air.

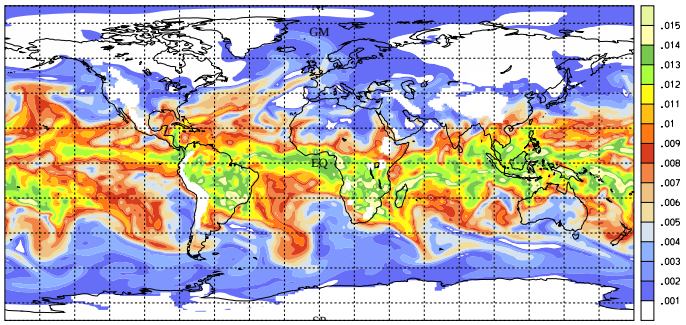

Figure 1: Distribution of the specific humidity [g/kg] at 850 hPa for Dec 1 2000 00 UTC, using the ERA40 dataset.

2. Water properties

The density of fresh-water is approximately given by the relationship

$$\rho = \rho_*(1 - \alpha_* \Theta_*^2)$$

(a) What is the percentage change in density for a 10 K change in Θ_*?

(b) Explain your result in terms of scale in one sentence.

(Hint: $\Delta\rho/\rho =$?. Compare page 14 and 15 in the lecture notes).

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3. Air properties

Assume a vertical column of air is

- isothermal (i.e. at uniform temperature)
- hydrostatic equilibrium
- and behaves like an ideal gas

Show that the variation of pressure with height satisfies the relationship

$$p(z) = p_0 \, e^{-\frac{g}{RT}z}$$

Also estimate:

(a) At what hight will the pressure have decreased to $(1/e)$ of its value at the surface? Assume a typical value for the mean temperature.

(b) What is the typical pressure difference between Jungfraujoch and Interlaken? Make an assumption for the mean temperature.

(c) The change in the value of the answer to (b) if the mean temperature in the layer changes by 15 K.

4. Viscosity

Viscosity is a measure of the resistance of a fluid to deformation under shear stress. Water and most gases, are known as Newtonian fluids, which means that the shear stress is proportional to the strain rate.

(a) Write down and explain the definition of the dynamic and kinematic viscosity.

(b) By increasing the temperature, the viscosity of air also increases but the viscosity of water decreases (compare Fig. 2.6 in lecture notes). Give a short explanation.