Mixing and Convection

Slantwise displacement

- Occurs in the atmosphere when a horizontal temperature gradient exists.
- If displacement occurs over large enough area then Coriolis force comes into play.
- Assume that isentropic surfaces (constant Θ) are tilted.
- Suppose parcel of air at point A is in equilibrium with the environment (same T, Θ, p, u, v).
- Next suppose parcel is slantwise displaced to B.

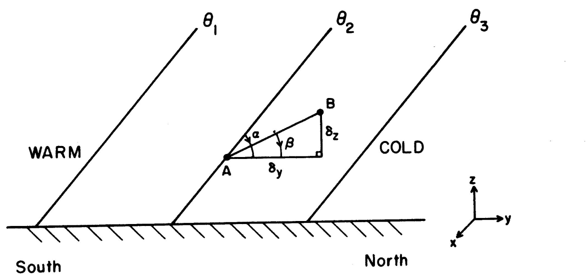

FIG. 3.7. Assessment of parcel stability for slantwise displacement from an equilibrium condition.

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- If no condensation occurs, Θ is conserved and the temperature of the parcel $T(y,z)$ at B is:

$$T + \left(\frac{dT}{dp}\right) dp = T + \frac{\kappa T}{p}\left(\frac{\partial p}{\partial y}\delta y + \frac{\partial p}{\partial z}\delta z\right) \quad (1)$$

- The ambient temperature ($\Theta \neq const.$) at B is given by:

$$T + \frac{\partial T}{\partial y}\delta y + \frac{\partial T}{\partial z}\delta z = T + \left(\frac{T}{\Theta}\frac{\partial \Theta}{\partial y} + \frac{\kappa T}{p}\frac{\partial p}{\partial y}\right)\delta y \quad (2)$$

$$+ \left(\frac{T}{\Theta}\frac{\partial \Theta}{\partial z} + \frac{\kappa T}{p}\frac{\partial p}{\partial z}\right)\delta z \quad (3)$$

- The excess temperature of the displaced parcel over the ambient air is:

$$\Delta T = -T\left(\frac{1}{\Theta}\frac{\partial \Theta}{\partial z}\delta z + \frac{1}{\Theta}\frac{\partial \Theta}{\partial y}\delta y\right) \quad (4)$$

- The buoyancy force on the displaced parcel is:

$$F_B = g\frac{\Delta T}{T} = -g\left(\frac{1}{\Theta}\frac{\partial \Theta}{\partial z}\delta z + \frac{1}{\Theta}\frac{\partial \Theta}{\partial y}\delta y\right) \quad (5)$$

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- In addition to the buoyancy force, the parcel is subject to a horizontal restoring force because the Coriolis and horizontal pressure forces acting on the parcel are out of balance at the new position.
- If the parcel moves from A to B in time δt , the Coriolis force changes the x-component of its velocity by an amount $\Delta v = f\ v\ \delta t = f\ \delta y$. At its new position, the Coriolis force CF_x on the parcel is therefore increased by the amount $\Delta CF_x = f\Delta v = f^2\delta y$.
- The tilt of the isentropic surfaces indicates that the horizontal pressure gradient force must vary with height. From the thermal wind equation, the change of the pressure force from A to B is given by:
- $$\frac{\partial}{\partial y}\left(-\frac{1}{\rho}\frac{\partial p}{\partial y}\right)\delta y + \frac{\partial}{\partial z}\left(-\frac{1}{\rho}\frac{\partial p}{\partial y}\right)\delta z = f\frac{\partial u_g}{\partial y}\delta y + f\frac{\partial u_g}{\partial z}\delta z \quad (6)$$

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Generalized equation for parcel displacement

- Because the parcel is in equilibrium at A, the net horizontal restoring force F_H at B is given by the difference between the incremental changes in the Coriolis force + the horizontal pressure gradient force:
- $$F_H = f\left[\frac{\partial u_g}{\partial z}\delta z - \left(f - \frac{\partial u_g}{\partial y}\right)\delta y\right] \quad (7)$$
- Thus, the equation of motion of the parcel along its direction of displacement, with distance denoted by Δ, is therefore:
- $$\frac{d^2\Delta}{dt^2} = F_B sin\beta + F_H cos\beta \quad (8)$$

$$= -g\left[\frac{1}{\Theta}\frac{\partial \Theta}{\partial z}\delta z + \frac{1}{\Theta}\frac{\partial \Theta}{\partial y}\delta y\right] sin\beta \quad (9)$$

$$+f\left[\frac{\partial u_g}{\partial z}\delta z - \left(f - \frac{\partial u_g}{\partial y}\right)\delta y\right] cos\beta \quad (10)$$

Generalized equation for parcel displacement

$$\frac{d^2\Delta}{dt^2} = -g\left[\frac{1}{\Theta}\frac{\partial\Theta}{\partial z}\delta z + \frac{1}{\Theta}\frac{\partial\Theta}{\partial y}\delta y\right] sin\beta \quad (11)$$

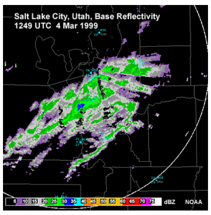
$$+f\left[\frac{\partial u_g}{\partial z}\delta z - \left(f - \frac{\partial u_g}{\partial y}\right)\delta y\right] cos\beta \quad (12)$$

- left hand side (LHS): acceleration of the air parcel
- first term on RHS: buoyancy force
- second term on RHS: pressure gradient force, u_g geostrophic wind.
- for δ y = 0 and $\beta = 90$ °, the buoyancy force, as discussed before, is the only force left:

$$F_B = \frac{d^2z}{dt} = g\left(\frac{T - T'}{T'}\right) = -\frac{g}{\Theta}\left(\frac{\partial\Theta}{\partial z}\right) z \quad (13)$$

Symmetric instability

- A type of slantwise instability, that occurs if the parcel is displaced along an isentropic surface, so that buoyancy force vanishes and the only restoring force is F_H.
- Symmetric instability can be responsible for the mesoscale banded structure of precipitation associated with midlatitude frontal systems.



Source: *http : //meted.ucar.edu/mesoprim/bandedprecip/print.htm#*4.4

Baroclinic instability

- another type of slantwise instability, which occurs if only the generalized buoyancy force is included.
- $$\frac{d^2\Delta}{dt^2} = -g\left(\frac{1}{\Theta}\frac{\partial\Theta}{\partial z}\right)\delta y \; sin\beta\left[\frac{\delta z}{\delta y} - \left(-\frac{\frac{\partial\Theta}{\partial y}}{\frac{\partial\Theta}{\partial z}}\right)\right] \quad (14)$$
- first term in brackets: slope of the air parcel displacement
- second term: slope of the isentropic surface.

Define stability in statically stable atmosphere ($\frac{\partial\Theta}{\partial z} > 0$):

- slope of isentropic surface < slope of parcel displacement $\rightarrow$ stable
- slope of isentropic surface = slope of parcel displacement $\rightarrow$ neutral
- slope of isentropic surface > slope of parcel displacement $\rightarrow$ unstable
- This instability mechanism, first investigated by Charney (1947) and Eady (1949) is often met in the atmosphere at midlatitudes
- It is firmly established that this kind of instability is responsible for the formation of midlatitude cyclones and the associated widespread cloud and precipitation.

Summary of instabilities

dry	absolute instability $\frac{d\Theta}{dz} < 0$ $-\frac{dT}{dz} > \Gamma_d$
conditional	conditional instability $\frac{d\Theta_{es}}{dz} < 0$ $\Gamma_m < -\frac{dT}{dz} < \Gamma_d$
convective (for layers)	convective instability (= potential instability) $\frac{d\Theta_e}{dz} < 0$

where Θ_{es} = saturation equivalent potential temperature

Isobaric mixing

- 2 air masses with M_1, T_1, q_1 and M_2, T_2, q_2. Mix them thoroughly at p=const. q of mixture = mass-weighted mean of individual q's:

$$q = \frac{M_1}{M_1 + M_2} q_1 + \frac{M_2}{M_1 + M_2} q_2 \tag{15}$$

and since $q \sim w$, the same holds for w.

- If no heat is gained or lost, then amount of heat lost by warmer sample = heat gained by colder sample:

$$M_1(c_p + w_1 c_{pv})(T_1 - T) = M_2(c_p + w_2 c_{pv})(T - T_2) \tag{16}$$

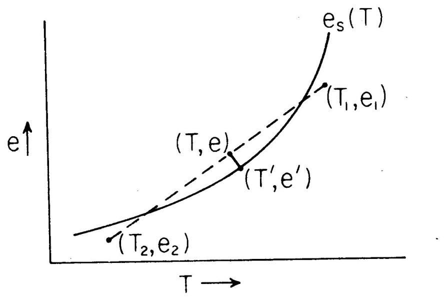
- since $w_1\, c_{pv} < c_p \Rightarrow$

$$M_1(T_1 - T) \sim M_2(T - T_2) \tag{17}$$

or

$$T = \frac{M_2 T_2 + M_1 T_1}{M_1 + M_2} \tag{18}$$

- Obtain total water $Q = q + w_l$ of the mixture in the same way.



- If condensation occurs, w is decreased, air is warmed by:

$$dq = -Ldw = c_p dT \; (1^{st} \, law \; with \; -\alpha dp = 0) \tag{19}$$

- with $w = \epsilon \frac{e}{p}$ and $dw = \epsilon \frac{de}{p}$ because p=const.:

$$\frac{de}{dT} = \frac{p}{\epsilon}\frac{dw}{dT} = -\frac{pc_p}{\epsilon L} \tag{20}$$

which is the slope of the line $(T, e) - (T', e')$ that describes the isobaric condensation process.
- Intersection of this line with e_s defines (T', e') of the mixture of the air mass after condensation

Adiabatic mixing

- Two air samples with initially different p's are thoroughly mixed after adiabatically being brought to the same p.
- When column of air is thoroughly mixed, q will tend to a constant value throughout:

$$q_m = \frac{1}{M}\int_{z_1}^{z_2} \rho q dz = \frac{1}{\Delta p}\int_{p_2}^{p_1} q dp \tag{21}$$

where $M = \int_{z_1}^{z_2} \rho dz$ = mass of column.
- Likewise for the potential temperature of the mixture:

$$\Theta_m = \frac{1}{\Delta p}\int_{p_2}^{p_1} \Theta dp \tag{22}$$

- After thorough mixing the lapse rate in a vertical column thus approaches the dry adiabatic lapse rate and the mixing ratio approaches a constant value.

Convective condensation level = CCL

- Vertical mixing within a column of air because of solar heating of the surface.
- Heat is transferred by conduction from surface to air layer in contact with it ⇒ strong lapse rate
- If lapse rate becomes superadiabatic any small disturbance will lead to vertical motion ⇒ general mixing and overturning
- Temperature profile then approaches the dry adiabat
- w is constant in mixed layer.
- If strong heating continues, heat will be convected upward ⇒ raising Θ throughout layer ($T_o \rightarrow T_1$)
- More heating ⇒ Θ higher, w constant up to C
- Heat added by convection: area (T_o, T_2, C).
- Only little extra heating necessary to reach CCL

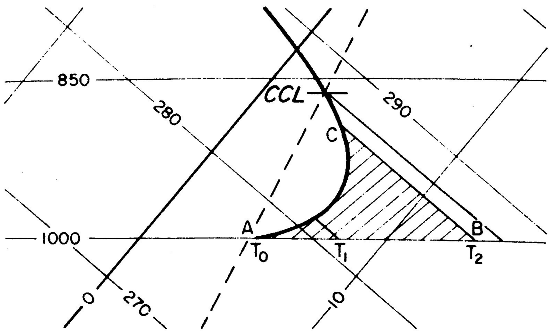


FIG. 4.2. Illustration of convective condensation level.

- If air is conditionally unstable above CCL ⇒ air continues to rise along pseudoadiabat
- In contrast to CCL, LCL depends only on surface properties; it does not account for any vertical mixing.
- If Cu are observed CCL ∼ LCL because air below cloud base is well mixed (i.e. w=const. and ambient lapse rate γ is the dry adiabatic one).
- In reality, heating often causes more evaporation from the Earth's surface ⇒ w higher ⇒ CCL lower

Convection: elementary parcel theory

= vertical motions of elements of air

- arising from buoyant or mechanical forces
- atmosphere's way to provide efficient vertical transport of heat, mass and momentum
- Buoyant convection ⇒ Cu formation
- It represents a conversion of potential energy to kinetic energy, and is expected to occur whenever heating at the surface or cooling aloft creates an unstable air layer.
- Now switching from effect of convection on lapse rate to the sizes and shapes of buoyant elements, their velocities and their interactions with the surrounding air.

Buoyant force

- $$\frac{d^2z}{dt^2} = g\left(\frac{T - T'}{T'}\right) = gB \qquad (23)$$
  where T is temperature of the air parcel, T' the ambient temperature, and B denotes the buoyancy term.
- Define $U = \frac{dz}{dt}$ as vertical velocity. Then
  $$\frac{dU}{dt} = gB \leftrightarrow \frac{dU}{dt}dz = UdU = gBdz \qquad (24)$$
  $$U^2 = U_o^2 + 2g\int_{z_0}^{z} B(z)dz \qquad (25)$$
  where U is the velocity at height z and U_o is the velocity at z_o.
- The integral represents the area on a thermodynamic chart bounded by the ambient and the parcels temperature profile, from pressure $p(z_0)$ to pressure p(z).

- This area is proportional to the increase in kinetic energy of the buoyant parcel between z_0 and z ⇒ "positive" area of the sounding (convectively available potential energy = CAPE):
  $$U^2 = U_o^2 + 2\,\mathrm{CAPE} \qquad (26)$$
- The level where the temperature profile of the parcel crosses with the ambient again, is referred to as LNB (level of neutral buoyancy).
- Predicting the velocity this way is an overestimation, because
- Thus, U predicted that way is an expected upper limit for vertical velocity in buoyant convection.

Account for the burden of condensed water

- If condensed water is present in the parcel, in form of cloud droplets or precip, it exerts a downward force on the parcel equal to its weight.
- The buoyancy factor B then becomes:

$$B = \frac{T}{T'} - (1 + \mu) \tag{27}$$

where μ [kg condensate/kg air] = mixing ratio of the condensate

- For adiabatic expansion with no mixing, and neglecting precipitation, $\mu = w_l$, the adiabatic LWC.
- The above expression assumes that there is no condensate in the ambient air around the thermal. However, if thermals were ascending through a cloud, then the general expression is:

$$B = \frac{T}{T'}(1 + \mu') - (1 + \mu) \tag{28}$$

where μ' is the mixing ratio of condensate in ambient air.

Compensating downward motion

- By requirement of mass continuity, air must descend somewhere to replace the volume vacated by an upward moving thermal.
- If the descending air is cloud free, it will be warmed at the dry adiabatic lapse rate. This will influence the temperature, and, hence buoyancy of the thermal.
- Focus on a horizontal level through which thermals ascend and ambient air descends, where A$'$ denotes the area where air is descending and A is the area of the thermals.
- The mass flux (MF) of upward-moving air through the level is ρUA [kg/s] and the downward MF is $\rho' U' A'$ (U = velocity of thermals).
- If the area of consideration is large enough then: MF$_u$ = MF$_d$ or:

$$\frac{A}{A'} = \frac{\rho' U'}{\rho U} \sim \frac{U'}{U} \tag{29}$$

assuming $\rho' \sim \rho$.

- $\Rightarrow$ smaller updrafts have higher velocities
- Further assume that ascending air follows pseudoadiabatic lapse rate, while descending air follows dry adiabatic lapse rate.
- Thus, after a short time dt, the air arriving from level below will have a temperature given by T$_o$ + $(\gamma - \Gamma_s)$ U dt, where T$_o$ is the initial temperature at that level. Γ_S denotes the pseudoadiabatic lapse and γ the ambient lapse rate.
- Air arriving from above has temperature: T$_o$ + $(\Gamma - \gamma)$ U$'$ dt.
- The situation is unstable when this temperature is less than the temperature of the thermal, i.e.:

$$(\gamma - \Gamma_s)U > (\Gamma - \gamma)U' \tag{30}$$

$$\leftrightarrow (\gamma - \Gamma_s)A' > (\Gamma - \gamma)A \tag{31}$$

- in the limit as A goes to zero, this is equivalent to $\gamma > \Gamma_s$ (unstable).

- The neutral case arises when

$$\frac{\gamma - \Gamma_s}{\Gamma - \gamma} = \frac{A}{A'} \quad (32)$$

- if $\frac{A}{A'} > 0$ (that is thermals are not negligible in size) this equation can only be satisfied if $\gamma > \Gamma_s$ (what was unstable before is now neutral).
- $\Rightarrow$ the ambient lapse rate must be steeper for instabilities to occur when compensating downward motions are taken into account

Dilution by mixing

- When a buoyant element ascends, some mixing takes place through the boundaries. Since the ambient air is generally cooler and drier than the buoyant element, mixing will reduce both the buoyancy of the thermal and lower its mixing ratio (entrainment).
- Account for entrainment by considering heat exchange between cloudy air and entrained air.
- Denote m as mass of cloudy air, which consists of dry air, water vapor and condensed water. Entrain mass dm through lateral sides as the cloud ascends through height dz.
- Heat required to warm the entrained air is:

$$dQ_1 = c_p(T - T^{'})dm \quad (33)$$

where temperature of cloudy air $= T$, of ambient air $= T^{'}$ and the heat contents of the vapor and the condensate are neglected.

- Next assume that just enough condensate evaporates to saturate the mixture. The heat required is:

$$dQ_2 = L(w_s - w^{'})dm \quad (34)$$

where w$^{'}$ is the mixing ratio of entrained air.

- Condensation occurs during the ascent, gaining latent heat:

$$dQ_3 = -mLdw_s \quad (35)$$

i.e. the parcel loses heats $dQ_1 + dQ_2$ but gains dQ_3.

- From 1. law of thermodyn:

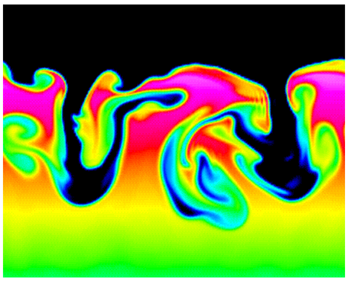
$$m(c_p dT - R_d T\frac{dp}{p}) = -dQ_1 - dQ_2 + dQ_3 \quad (36)$$

- divide both sides by mc_pT:

$$\left(\frac{dT}{T} - \frac{R_d}{c_p}\frac{dp}{p}\right) = \frac{-dQ_1 - dQ_2 + dQ_3}{mc_pT} \quad (37)$$

$$\frac{d\Theta}{\Theta} = -\frac{L}{c_pT}dw_s - \left[B + \frac{L}{c_pT}(w_s - w^{'})\right]\frac{dm}{m} \quad (38)$$

- without entrainment (dm = 0) we get back the change in Θ due to the pseudoadiabatic process.
- Because the bracketed term is always positive in cases of interest, the above equation implies that the temperature falls off at a faster rate with entrainment, i.e. buoyancy is impaired by entrainment.
- Alternative to lateral mixing is mixing of dry environmental air from just above cloud top.
- Turbulence draws parcel of ambient air into the cloud, causing the evaporation of some cloud droplets. This will chill the parcel, reducing its buoyancy, leading to a downdraft.
- The cumulative effect of many such penetrative downdrafts from cloud top will be to cool and dry the cloud, especially in its upper regions.
- It is said to be the mechanism for the break-up of stratus clouds into stratocumulus when going from subtropics to tropics.



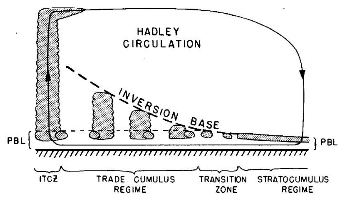
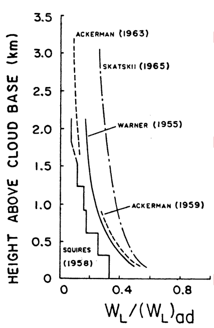


FIG. 5. A schematic illustration of the role of CIFKU in determining the tropical and subtropical distributions of cloudiness. Details are given in the text.

adiabatic liquid water content (LWC)



- In most cases LWC < LWC_{ad} → parcel theory
- LWC ∼ LWC_{ad}:
  1) near cloud base
  2) within undiluted "adiabatic" pockets
- LWC/LWC_{ad} ∼ 0.1 and 0.6

Fig. 2-22: Ratio of the observed mean liquid water content at a given height above cloud base to the adiabatic value, for non-precipitating clouds. (From Warner, 1970a; by courtesy of Am. Meteor. Soc., and the author.)

→ cloud as a whole has significantly less water than adiabatic value, because of entrainment

Aerodynamic resistance: thermals and plumes

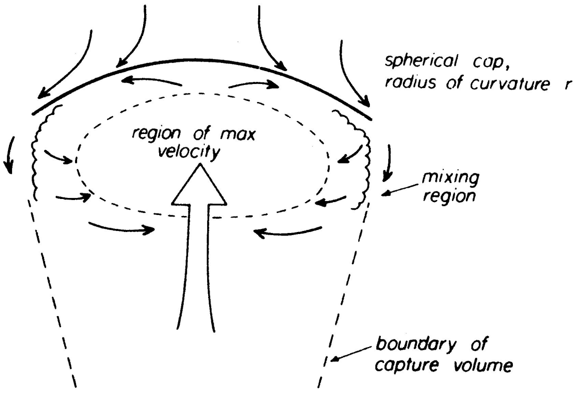


FIG. 4.3. Structure of a convective bubble.

- This is an idealized thermal (spherical cap, radius of curvature r), based on laboratory studies
- Resembles atmospheric thermals which appear as "turrets" or protuberances of cumuli. They are said to be shape-preserving.
- Thus, the vertical velocity of a bubble depends on its size and buoyancy according to:

$$u = c\sqrt{g\overline{B}r} \tag{39}$$

where u is the upward velocity of the cap, $\overline{B}$ is the average buoyancy factor across the bubble, r is the radius of the cap, and c=1.2 is given from experiments

- In the atmosphere, however, cumuli are more complicated than these elementary bubbles.
- Their velocity is related to the stability of the air and the size and state of development of the cloud as a whole, and cannot be predicted for all clouds and for all occasions with the above equation.

Another approach is taken for an idealized conical plume, where profiles of velocity and buoyancy across the plume are identical (see right).

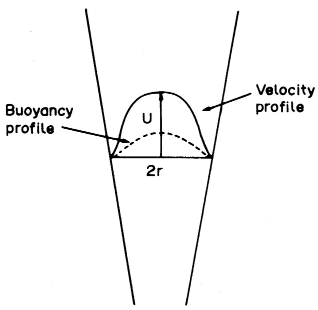


FIG. 4.4. Vertical cross-section of plume.