

## Atmosphärenphysik: Formation and growth of ice crystals



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## Formation and growth of ice crystals



- Once a cloud extends to altitudes where  $T < 0^\circ \text{C}$ , ice crystals (IC) may form either by freezing of a CD or direct deposition of vapor to the solid phase. Both are nucleation processes and in principle homogeneous and heterogeneous nucleation are possible.
- A newly formed IC in a cloud of liquid droplets is in a favored environment to grow rapidly by diffusion. The vapor in the cloud is essentially saturated with respect to water and hence supersaturated with respect to ice.
- In only a few minutes such an IC can grow to many tens of microns.  $\rightarrow$  falls with a velocity  $\sim$  a few tens cm/s. It may reach the ground as an individual IC, collide with supercooled CD to form a rimed crystal, or with other crystals to form a crystal aggregate or snowflake.
- Growth processes are thus the same as for a CD, namely diffusion followed by coagulation. For IC, diffusional growth is more significant than for CD, because of the difference in saturation vapor pressure over water and ice.

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## Nucleation of the ice phase



- Homogeneous freezing of a pure liquid drop occurs when statistical fluctuations of the molecule arrangement of water produce a stable, ice-like structure that can serve as an ice nucleus (IN).
- Homogeneous nucleation depends on the size of the stable nucleus and the probability of occurrence of embryonic IC by random rearrangement of water molecules.
- These quantities depend on the surface free energy of a crystal/liquid interface (analogous to surface tension at a liquid/vapor surface). The value of the surface free energy is not known accurately ( $\sim 0.02 \text{ N/m}$ ).

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## Nucleation of the ice phase



- Consistent with experimental data on freezing of pure water, CD  $< 5 \mu\text{m}$  will freeze spontaneously at  $T < -40^\circ\text{C}$ . Larger droplets freeze at slightly warmer temperature.
- Some liquid in clouds as cold as  $-20^\circ\text{C}$  is not at all uncommon. Note that in bulk water a single nucleation event anywhere suffices to cause the entire mass to freeze. In a cloud, every CD has to have a nucleation event.
- Homogeneous deposition occurs when vapor molecules form a stable ice embryo by chance collisions. Theory predicts that homogeneous nucleation of deposition should only occur for extreme conditions of supersaturation which never occur in the atmosphere. This is confirmed by experiments, because CD always nucleate homogeneously before the supersaturation reaches the high values required for ice nucleation. As these CD freeze spontaneously at  $T < -40^\circ\text{C}$ , it is impossible to recognize IC that might be formed by deposition.

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## Nucleation of the ice phase



- IC appear in cloud in appreciable numbers when  $T < -15^\circ\text{C}$ , signifying heterogeneous nucleation. Water in contact with most materials freezes at  $T > -40^\circ\text{C}$ , and deposition occurs on most surfaces at supersaturation and supercooling less than the homogeneous nucleation values  $\rightarrow$  the nucleation of ice in supercooled water of a supersaturation environment is aided by the presence of **foreign surfaces** of suspended particles.
- The foreign material provides a surface or **substrate** on which water molecules can impinge, stick, bond together, and form aggregates with an ice-like structure. The larger the aggregate, the more likely it is to be stable and continue to exist.
- The probability of heterogeneous nucleation of freezing or deposition depends strongly on the properties of the substrate material as well as on the **supercooling** and **supersaturation**. The more **tightly-bound the water molecules** are to the substrate, the greater will be the probability of ice nucleation. The more closely the **crystal structure** of the substrate resembles that of an IC plane, it will increase the chances of ice nucleation.

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## Nucleation of the ice phase



- Supercooled clouds in the atmosphere develop and exist in the presence of vast numbers of aerosol particles, a small fraction of which serve as IN at temperature considerably warmer than the  $-40^\circ\text{C}$  threshold for homogeneous freezing:
- Ice may form directly from the vapor phase on suitable deposition nuclei.
- three modes of activation are recognized for freezing nuclei:
  - a) IN serve first as centers for condensation, then as freezing nuclei
  - b) IN promote freezing at the instant they come into contact with a supercooled droplet
  - c) IN cause freezing after becoming embedded in a CD.
- A given IN might nucleate ice in different ways, depending on the ambient conditions and its history in the cloud.

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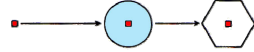
## Ice Nucleation Mechanisms



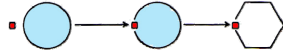
HETEROGENEOUS  
DEPOSITION



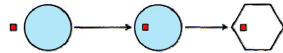
CONDENSATION  
FOLLOWED BY  
FREEZING



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## Nucleation of the ice phase



- The relative importance of different freezing modes in the atmosphere has not been established. It is also difficult to distinguish between deposition and freezing nucleation when ice nucleates on an insoluble surface in an environment supersaturation with respect to water (a) and (b).
- Even conditions below water saturation nucleation need not imply deposition, because the nuclei may contain soluble components. The soluble material may nucleate the liquid phase below water saturation and enable the insoluble material to nucleate ice by freezing. Because of the confusion between mechanisms, one often speaks of "ice nucleation" as the phenomenon instead of being more specific. Likewise, the atmospheric particles serving as nucleation centers can most safely be referred to as "ice nuclei".
- I.e. ~ ice nucleation is very uncertain both theoretically and experimentally

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## Experiments on heterogeneous ice nucleation



- The nucleating properties of small particles are studied by introducing them into **cloud chambers** with controlled supercooling and supersaturation. The conditions are noted where the onset of nucleation occurs. (IC are usually discernable even in the presence of liquid by the scintillation of light scattered from a strong beam). It's not possible to distinguish between a deposition event and a condensation event followed by freezing.
- Other experiments consist of adding finely divided material to supercooled, purified water and noting the threshold temperature for freezing. The table below summarizes the threshold ice-forming temperatures of certain pure and natural substances.

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## Experiments on heterogeneous ice nucleation



TABLE 9.1. Temperatures at which different substances nucleate ice. (From Houghton, 1985)

Substance	Crystal lattice dimension		Temperature to nucleate ice (°C)	Comments
	a axis (Å)	c axis (Å)		
<b>Pure substances</b>				
Ice	4.52	7.36	0	—
AgI	4.39	7.49	-4	Insoluble
PbI <sub>2</sub>	4.54	6.96	-6	Slightly soluble
CuS	3.80	16.43	-7	Insoluble
CuO	4.65	5.11	-7	Insoluble
HgI <sub>2</sub>	4.36	12.34	-8	Insoluble
Ag <sub>2</sub> S	4.20	9.50	-8	Insoluble
CdI <sub>2</sub>	4.24	6.84	-12	Soluble
I <sub>2</sub>	4.78	9.77	-12	Soluble
<b>Minerals</b>				
Vaterite	4.12	8.56	-7	—
Kaolinite	5.16	7.38	-9	(Silicate)
Volcanic ash	—	—	-13	—
Hallopyrite	5.16	10.1	-13	—
Vermiculite	5.34	28.9	-15	—
Cinnabar	4.14	9.49	-16	—
<b>Organic materials</b>				
Testosterone	14.73	11.01	-2	—
Cholesterol	14.0	37.8	-2	—
Metalddehyde	—	—	-5	—
β-Naphthol	8.09	17.8	-8.5	—
Phloroglucinol	—	—	-9.4	—
Bacterium	—	—	-2.6	(Bacteria in leaf mold)
<i>Pseudomonas</i>	—	—	—	—
<i>Syringae</i>	—	—	—	—

Silver iodide matches ice closest in structure and also has a relatively warm nucleating temperature. However, lattice dimensions are not the only factor determining nucleating ability, as several organic materials do not have a well-defined crystal structure. Some combinations of lattice matching, molecular binding, and low interfacial energy with ice accounts for the nucleating ability of a substance. Still an open field.....

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## Atmospheric ice nuclei



- .. are studied in cloud chambers and on filter systems, into which samples of air are drawn.
- In a cloud chamber the sample is cooled down to a controlled temperature and a cloud is formed by adding sufficient water vapor. Most time, an optical system is used to count the no of ice crystals that form as a function of degree of supercooling. However, no information about nucleation mode and nucleus size can be retrieved that way.
- The second method consists of collecting aerosols by drawing the air sample through filter paper with known pore sizes. The particulates trapped on the filters are then introduced to an environment suitable for IC development and observations are made of the number of crystals that form on a substrate. That way, information about nucleus size can be obtained, but not about nucleation mode.

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## Atmospheric ice nuclei



- Reported IC concentration at temperatures from -15°C to -20°C range over many orders of magnitude. As the supercooling increases, so does the nucleus count at usually a rapid rate. IN concentration also increases with supersaturation. Furthermore, there is evidence that some nucleation events do not occur immediately, but require a long exposure of the nucleus to supercooled conditions. Accordingly, only a fraction of the IN actually present in an air sample may be activated during the time of an experiment. IN content of the air is a highly variable quantity.
- Fletcher (1962), reviewing the data to that time, gave as a typical concentration **1 nucleus per liter of air at T = -20°C**, increasing by a factor of 10 for each 4°C of additional cooling.
- This exponential dependence of IN on supercooling is accepted as typical, but it is recognized that the count at any given location and time can be at least an order of magnitude above or below.

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## Atmospheric ice nuclei



- Taking  $10^4 \text{ cm}^{-3}$  as a typical AP concentration, we see that 1 nucleus per liter active at  $-20^\circ\text{C}$  is only 1 IN in  $10^7$ . Separating and identifying such rare particles is not an easy task. Much evidence points to **clay minerals**, especially **kaolinite** as a major component of atmospheric IN. This is a common material found in many soil types. Snowflakes analysis yields that they contain kaolinite with sizes ranging from  $0.1 \mu\text{m}$  to  $4 \mu\text{m}$ . What is not clear is how kaolinite could explain the occurrence of ice in clouds warmer than  $-9^\circ\text{C}$ , as sometimes observed.
- Another source of IN has been revealed by the discovery that the **bacteria** in decaying plant leaf material can be effective nucleants at warm temperature. Bacterium *pseudomonas syringae* serves as IN at  $-1.3^\circ\text{C}$ , although its nucleating ability is a rare and changeable property. Thus, more work needs to be done here.

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## Atmospheric ice nuclei



- **Meteoritic material** is another source, found, because of apparent correlation between extreme rainfall events and meteor showers and more recently from experiments. However, more evidence points to a terrestrial source at ground, as even at the South Pole, the particulates in snow flakes are found to be clay minerals. Also IN tend to decrease with altitude over continents and at coastal sites measurements indicate more nuclei in air from trajectories over land.
- In summary, while supersaturation with respect to water exceeding 1% are extremely rare, supercooling of liquid water to  $-15^\circ\text{C}$  or colder is not uncommon. Also, probably more than one material serves as IN depending on temperature, humidity and distance from sources.

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## The ice phase in clouds



- Occurrence of IC in clouds is related to cloud type, temperature and cloud age. Overall observations confirm that the colder cloud top temperature, the greater is the likelihood that some IC are present along with the supercooled CD. Although essentially no clouds with tops as cold as  $-20^\circ\text{C}$  are ice-free, ice anywhere in a cloud is unlikely unless the top extends to  $-5^\circ\text{C}$  or colder.
- Ice is more common in decaying Cu than in newly developing clouds, and is probably more common in stratiform clouds than in Cu with the same cloud top temperature.
- Measured concentration of IC range from the lower limit of detection (0.01/l) to 100/l. Concentrations are high in Ci, and still higher in ice fogs, which develop under extremely cold arctic conditions. The most perplexing question in ice microphysics is to explain the vast discrepancies, something accounting to a factor of  $10^4$ , between observed crystal concentration and the measured concentration of IN in ambient air.

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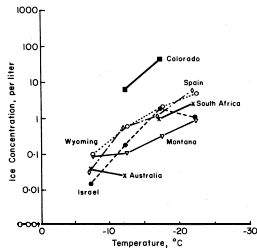
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## The ice phase in clouds



The first IC to appear in a cloud must form on IN (exception may be cirrus, which form at such cold temperature that homogeneous freezing may occur as soon as the liquid phase appears). Additional crystals may then be produced by secondary processes in which the primary crystals are **"multiplied"**.



The Figure here shows the observed IC concentration in clouds where secondary processes of ice formation are thought to be unimportant (i.e. IC ~ IN). Mossop (1985) explained that IC usually agree with IN to within a factor of 10 in clouds that contain no CD larger > 25  $\mu$ m diameter at temperature warmer than -10°C. For other cloud types, the discrepancy is larger.

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## The ice phase in clouds



- **Secondary IC:** (a) **fracture** of IC, and (b) **splintering** of freezing drops. Often IC fragments are included with falling snow. These are probably produced when dense graupel particle overtake and collide with fragile, slower-falling dendritic crystals. Not much is known about (a).
- Another mechanism of ice multiplication, thought to be very effective in the right conditions, is the production of secondary particles when supercooled drops of the appropriate size and temperature are captured by graupel particles.

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## The ice phase in clouds



- **Hallett-Mossop process:** small particles are ejected by drops larger than 25  $\mu$ m in diameter when they freeze on to an ice substrate at temperature between -3 and -8°C. At temperature higher than -3°C drops tend to spread over the ice surface instead of freezing as discrete drops. At temp colder than -8°C, the freezing is thought to proceed so rapidly, starting with an outer shell of ice, that disruption does not occur. Drops smaller than 25  $\mu$ m in diameter probably freeze too rapidly to be disrupted. This mechanism of secondary particle production, sometimes called **rime-splintering**, may account for high concentration of IC that are sometimes observed in maritime Cu with temperature no colder than -10°C.
- **Summary:** no doubt that IC are produced by secondary processes, but we don't know exactly how.

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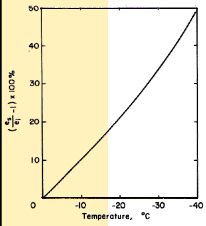
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## Diffusional growth of IC



This figure shows **supersaturation**  $S_s$  for water saturation. Water-saturated cloud has high supersaturation with respect to ice and is a favorable environment for rapid growth by diffusion and deposition. The environment will remain favorable for crystal growth as long as CD are available to evaporate and maintain the vapor pressure at equilibrium relative to water. If the CD eventually disappear, by evaporation or freezing, the saturation ratio will decrease to equilibrium relative to ice.

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## Diffusional growth of IC



- Growth of IC by **diffusion** is analogous to that of a CD but with a complication because of the nonspherical shape of the IC.
- Maxwell in his theory of the wet-bulb thermometer solved the equations of heat and mass transfer by drawing on the analogy between the diffusion equation and equations in electrostatics that describe the distribution of potential around a charged conductor. The electrostatic analogy is the starting point of the theory of IC growth by diffusion.

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## Diffusional growth of IC



- From this analogy between the electric field's potential and the water vapor density  $\rho_v$ , as well as the physical meaning of

$$-D\nabla\rho_v$$

as the vapor mass flux vector (by diffusion with coefficient  $D$ ) at the IC surface  $S$ , it can be derived that the growth rate of the particle is given by:

$$\frac{dm}{dt} = \int_S D \Delta \rho_v \cdot n dS = 4\pi C D (\rho_v - \rho_{v,r})$$

where  $C$  denotes the electrical capacitance, with length units, a function of the size and shape of IC. For a sphere  $C=r$  and equation above reduces to the growth equation for water droplets. For a circular disk of radius  $r$ , which can be used as an approximation for plate-type IC,  $C = 2r/\pi$ .

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## Diffusional growth of IC



- Ice needles may be approximated by the formula for a prolate (länglich) spheroid of major and minor semi-axes  $a$  and  $b$ , for which

$$C = \frac{A}{\ln\left[\frac{a+A}{b}\right]}$$

- where  $A=(a^2 - b^2)^{0.5}$ . For an oblate (abgeplattet) spheroid

$$C = \frac{a\varepsilon}{\arcsin \varepsilon}$$

- where the ellipticity  $\varepsilon = (1 - b^2/a^2)^{0.5}$ .

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## Diffusional growth of IC



- Actual IC have more complex shapes than the spheres, disks, and ellipsoids for which these theoretical formulas apply. We approximate plane dendrites and plates by a circular disk of equal area. Likewise we approximate needles by long prolates.

- The ice crystal growth equation is analogous to that for CD:

$$\frac{dm}{dt} = \frac{4\pi C(S_i - 1)}{\left[\left(\frac{L_s}{R_v T} - 1\right) \frac{L_s}{KT} + \frac{R_v T}{e_i(T)D}\right]}$$

- here as for CD, kinetic effects and ventilation are neglected.  $K$  as in for the droplet growth is the thermal conductivity of air.

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## Diffusional growth of IC



- Those effects are not understood as well for IC as for CD. Vapor molecules cannot unite with an IC in any haphazard way, but must join up, molecule-by-molecule, in such a manner that the crystal pattern is maintained.

- Consequently it may be incorrect to identify  $\rho_{vr}$  with the equilibrium vapor density of ice, and in fact  $\rho_{vr}$  may not be the same over all points of the crystal surface. Because of these effects the rate of growth of an IC will tend to be slower than given by above equation. It is only 50% of that for small IC at temperature warmer  $-10^\circ$  C.

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## Some numbers



TABLE 7.1. Values of Dynamic Viscosity  $\mu$  and Coefficient of Thermal Conductivity  $K$ , of Air, and Coefficient of Diffusion  $D$  of Water Vapor in Air. (From Houghton, 1985)

$T(^{\circ}\text{C})$	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ )	$K$ ( $\text{J m}^{-1}\text{s}^{-1}\text{K}^{-1}$ )	$D$ ( $\text{m}^2\text{s}^{-1}$ )
-40	$1.512 \times 10^{-5}$	$2.07 \times 10^{-2}$	$1.62 \times 10^{-5}$
-30	$1.564 \times 10^{-5}$	$2.16 \times 10^{-2}$	$1.76 \times 10^{-5}$
-20	$1.616 \times 10^{-5}$	$2.24 \times 10^{-2}$	$1.91 \times 10^{-5}$
-10	$1.667 \times 10^{-5}$	$2.32 \times 10^{-2}$	$2.06 \times 10^{-5}$
0	$1.717 \times 10^{-5}$	$2.40 \times 10^{-2}$	$2.21 \times 10^{-5}$
10	$1.766 \times 10^{-5}$	$2.48 \times 10^{-2}$	$2.36 \times 10^{-5}$
20	$1.815 \times 10^{-5}$	$2.55 \times 10^{-2}$	$2.52 \times 10^{-5}$
30	$1.862 \times 10^{-5}$	$2.63 \times 10^{-2}$	$2.69 \times 10^{-5}$

Note: The tabulated values of  $D$  are for a pressure of 100 kPa. Because  $D$  is proportional to  $\mu/g$ , it follows that  $D$  is inversely proportional to pressure  $p$  for a given temperature. To obtain  $D$  for an arbitrary pressure  $p$  (kPa), the tabulated value must therefore be multiplied by  $(100/p)$ .

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## Diffusional growth of IC



- Growth rate depends on temperature and pressure

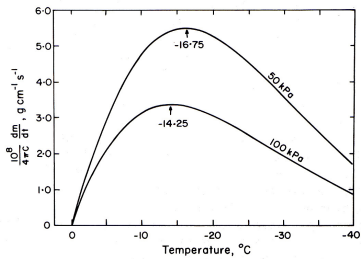


FIG. 9.4. Normalized ice crystal growth rate as a function of temperature. (Adapted from Byers, 1965.)

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## Diffusional growth of IC



- Ambient conditions determine not only growth rate, but also the form, or habit that a growing crystal takes. All of these forms are basically hexagonal structures, but with widely different axis ratios.

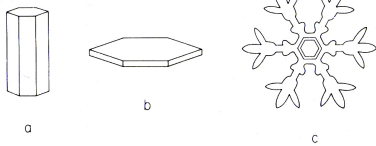


FIG. 9.5. Schematic representation of the main shapes of ice crystals: (a) column; (b) plate; (c) dendrite.

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## Diffusional growth of IC

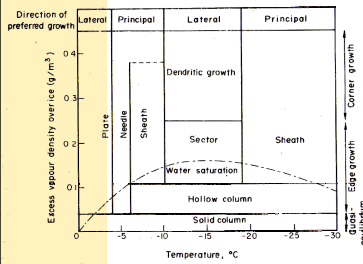


FIG. 9.6. Kobayashi's diagram of crystal habit as function of temperature and excess vapor density over ice saturation. (From Fletcher, 1962.)

As a growing crystal moves through a cloud its crystal habit will change according to the changing ambient conditions. Sector stars are formed when plates develop peripheral dendritic structure; capped columns arise when columns develop plates on their ends. The intricate stellar shapes which are often observed are variations on the dendritic form

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## Diffusional growth of IC



- Excess vapor density is at maximum at about  $-15^{\circ}\text{C}$  as is the temperature of maximum growth rate. The preferred crystal types in this growth region are seen to be **dendrites** and **sectors**.
- Diffusion alone cannot explain the different growth habits of ice or the transitions from one habit to another. A **molecular-kinetic approach** is required to explain these effects. Microscopic examinations of the surface of growing crystals show that molecules of water that impinge on the surface of the crystal are bound more strongly at the ledges than on the terraces. They fix themselves preferentially at the ledges, causing growth by the lateral motion of surface steps. The rate of growth is determined by the rates at which steps are generated and advance, and, these rates in turn are controlled by temperature and supersaturation. The main mechanism for the motion of steps is the diffusional movement of molecules across terrace sites up to and into ledge sites. These kinetic effects determine the crystal habit and tend to slow the rate of crystal growth from the value given by the continuum vapor diffusion theory.

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## Diffusional growth of IC



- Without explaining the shapes of crystals, the classical theory can nevertheless be used for approximate growth calculations if the crystal habit is specified.
- What is required is an analytical expression for the capacitance of the crystal and empirical relations between the dimensions of the crystal and its mass.

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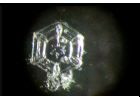
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## Further growth by accretion



- Strictly speaking, growth by **accretion** occurs when any large precipitation particle overtakes and captures a smaller one.
- However, in common usage it is reserved for capture of supercooled CD by an ice-phase precipitation particle. If CD freeze immediately on contact they form a coating of rime, leading to **rimed crystals** or graupel. If freezing is not immediate, denser structures are created, of which hail is an extreme example.
- In correspondence to coalescence (CD + larger drop or raindrop) we use the term **aggregation** for the clumping together of IC to form snow **flakes**.

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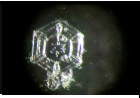
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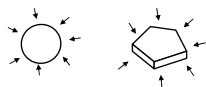
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## Summary of growth processes of precipitation particles



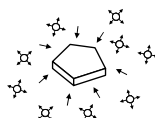
### Condensation, Sublimation      Coalescence, Aggregation



### Riming



### Findeisen - Bergeron




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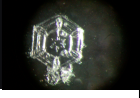
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## Further growth by accretion



- Difference in fall speed: fastest are graupel particles (which are not really crystals, but aggregates of frozen droplets). The rimed structures and dry snow flakes fall at about 1 m/s, but all pure crystal types fall slower than 1m/s. Empirical fall speed for **graupel**:

$$u = 343D^{0.6}$$

- where  $u$  is in cm/s and  $D$ , the diameter of the sphere which just circumscribes the particle, in cm. Empirical fall speed for snow flakes:

$$u = kD^n$$

- where  $D$  is the melted diameter. With  $D$  in cm and  $u$  fall speed in cm/s, Langleben found for dendrites  $k = 160$  and  $n = 0.3$ , and for columns and plates  $k = 234$  and  $n = 0.3$ .

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## Terminal fall velocity

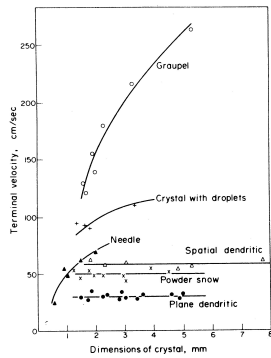


FIG. 9.7. Nakaya and Terada's measured terminal velocities of ice crystals. (From Fletcher, 1962.)

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## Further growth by accretion



- Mass and size of different forms of IC are usually related by empirical formulas of the form:

$$m = aD^b$$

- where

TABLE 9.2. Values of  $a$  and  $b$  in (9.7) for  $D$  in cm and  $m$  in g

Crystal type	$a$	$b$
Graupel	$6.5 \times 10^{-2}$	3
Thin hexagonal plate	$1.9 \times 10^{-2}$	3
Stellar crystal	$9.4 \times 10^{-4}$	2
Planar dendrite	$3.8 \times 10^{-4}$	2
Needle	$2.9 \times 10^{-5}$	1

- where  $D$  is the major linear dimension of the crystal in centimetres and  $m$  in g.

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## Further growth by accretion



- Collision efficiencies for accretion should be larger than for CD, because of lower fall speed. Coalescence efficiencies are approximately 1, because freezing is likely to occur on contact with supercooled CD. ( $E=1$  in ECHAM)
- $E$  for aggregation is less well understood. Indications are that open structures like dendrites are more likely to stick, given a collision, than crystals of other shape, and that sticking in any case is more likely at relatively warm temperatures. → judging from observed sizes of snowflakes as a function of temperature, it has been inferred that significant aggregation is possible only at temperature  $> -10^\circ\text{C}$ . ( $E_{\text{agg}} = e^{0.025(T-T^0)}$ ).

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## Further growth by accretion



- Growth equation, leading to graupel, analogous to rain drops:  
$$\frac{dm}{dt} = \bar{E}M\pi R^2 u(R)$$
 where  $M$  is cloud liquid water content
- similar for aggregation:  
$$\frac{dm}{dt} = \bar{E}M\pi R^2 \Delta u$$
 where  $M$  is frozen water density
- where  $\Delta u$  is the difference in fall speed of the snowflake and the IC, essentially a constant ( $u$  (snowflake)  $\sim 1$  m/s and  $u$ (IC)  $\sim 0.4 - 0.5$  m/s).

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## Further growth by accretion



- Those growth equations are only rough approximations for actual growth process, but fit observations of graupel and snowflakes well.
- Fundamentally snowflakes develop, because a few crystals, which formed and grew by diffusion, become larger than their neighbors, either by enhanced diffusional growth or by chance collisions with other IC or supercooled CD. Thereafter they're in a favorable position to grow by the sweep-out process.

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## IC process versus coalescence



- Condensation-diffusion alone cannot explain raindrop formation in realistic time. This process is more effective for IC than for CD, because the vapor in the cloud is often at equilibrium relative to water and hence supersaturated with respect to ice. Therefore light precipitation can occur as individual crystals, indicating that aggregation or accretion never took place (clear sky precipitation). Thus, some drizzle or light rain has its origin in unaggregated crystals, which melted before reaching the ground.

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