UMNW, Mathematik 1, Lösung Serie 6

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1

1.1

```
> datax := [seq(i/4,i=0..4)];
                         datax := [0, 1/4, 1/2, 3/4, 1]

> datay := evalf(map(exp,datax));
datay := [1., 1.284025417, 1.648721271, 2.117000017,
   2.718281828]
```

1.2

$$
\begin{aligned}
\sum_{k=1}^{n}(a+bx_k+cx_k^2-y_k)^2 &= \sum_{k=1}^{n} r_k^2 = R(a,b,c) \\
&\Rightarrow \\
\frac{\partial}{\partial a}R(a,b,c) &\overset{!}{=} 0 \\
\frac{\partial}{\partial b}R(a,b,c) &\overset{!}{=} 0 \\
\frac{\partial}{\partial c}R(a,b,c) &\overset{!}{=} 0
\end{aligned}
$$

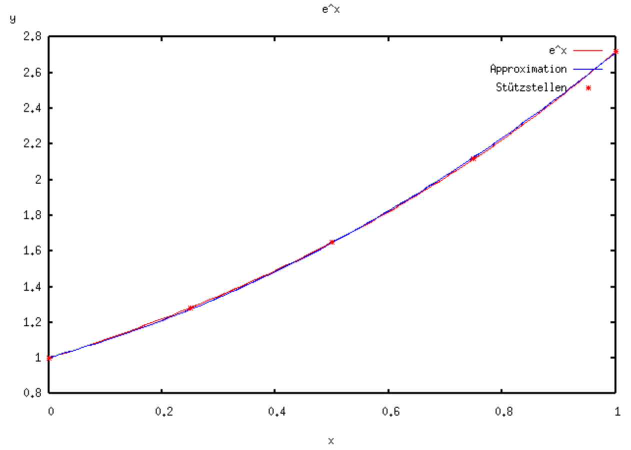
1.3

```
> with(stats): with(fit): with(plots):
> f := unapply(leastsquare[[x,y], y=a+b*x+c*x^2,{a,b,c}]\
> ([datax, datay]),x);
                                                          2
f := x -> y = 1.005140300 + .8642773549 x + .8435379451 x
```


1.4

```
> d:=map(x->evalf(abs(exp(x)-solve(f(x)))),datax);
d := [.005140300, .010094656, .000557808, .010838393,
   .005326228]
```

1.5

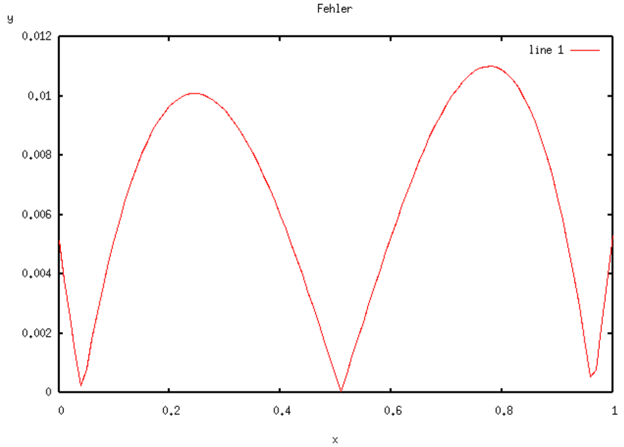


1.6

$$\max_{0\le x\le 1} |e^x - (1.005140300 + 0.8642773549x + 0.8435379451x^2)|$$

Ableiten und Nullstellen im Intervall $[0,1]$ suchen.

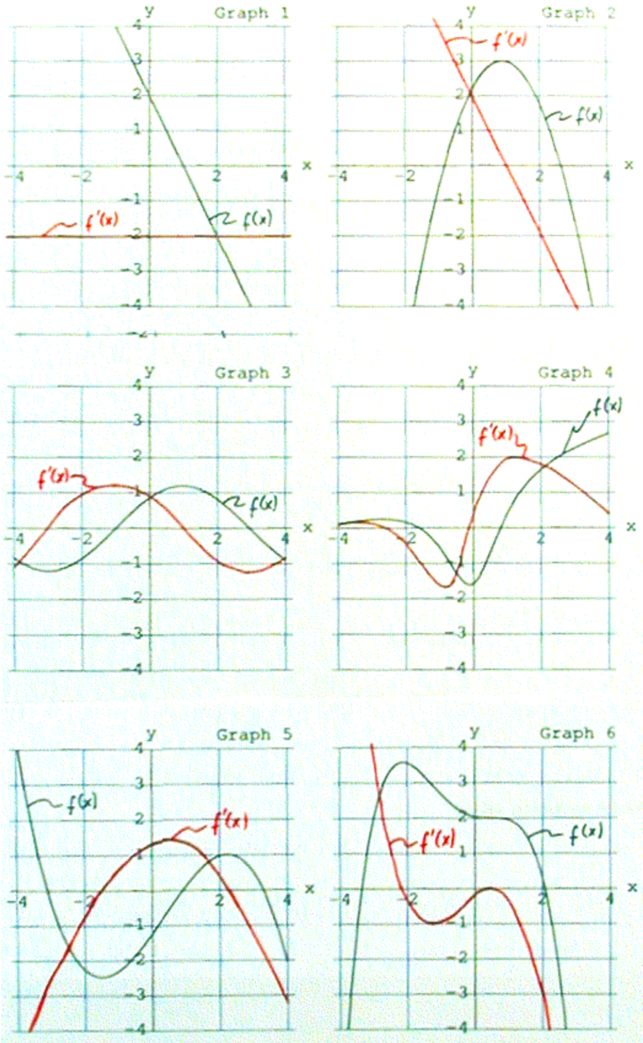
$$e^x - (0.8642773549 + 2\cdot 0.8435379451x) \overset{!}{=} 0$$




```
> f_1 := x -> y =exp(x); #wie kann man das ohne diesen\
 Murks machen? Also aus f := x -> y = ... wieder ein\
 f := ... machen?
                      f_1 := x -> y = exp(x)

> w_1 := fsolve(diff(f(x)-f_1(x),x),x=0..1);
                        w_1 := .2450253158

> w_2 := fsolve(diff(f(x)-f_1(x),x),x=0.3..1);
                        w_2 := .7774048026

> f(w_1)-f_1(w_1);
                         0 = -.010099696

> f(w_2)-f_1(w_2);
                          0 = .011014509
```

Der Fehler im Intervall $[0,1]$ ist also bei $x = 0.7774048026$ am grössten.

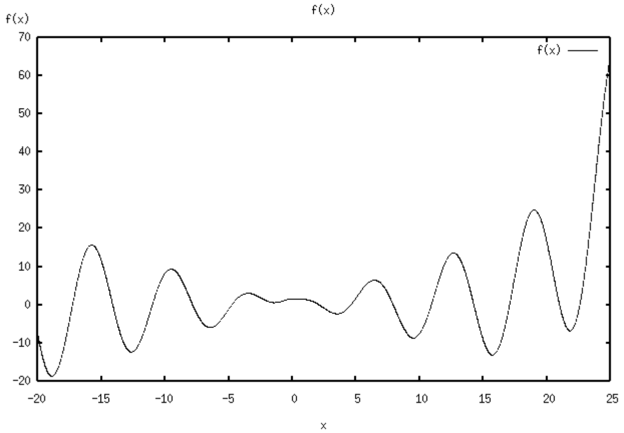
2



3

$$f(x) = \frac{2 + x^3 \cos x + 1.5^x}{2 + x^2}$$

3.1



3.2

$$\lim_{x\to\infty} \frac{2 + x^3 \cos x + 1.5^x}{2 + x^2} = \underbrace{\lim_{x\to\infty} \frac{2}{2+x^2}}_{=0} + \underbrace{\lim_{x\to\infty} \frac{x^3 \cos x}{2+x^2}}_{\to[-1,1]\cdot\infty} + \underbrace{\lim_{x\to\infty} \frac{1.5^x}{2+x^2}}_{\infty}$$

da jedoch gilt

$$|\lim_{x\to\infty} \frac{x^3 \cos x}{2+x^2}| < |\lim_{x\to\infty} \frac{1.5^x}{2+x^2}| \Rightarrow \lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to-\infty} \frac{2 + x^3 \cos x + 1.5^x}{2 + x^2} = \underbrace{\lim_{x\to-\infty} \frac{2}{2+x^2}}_{=0} + \underbrace{\lim_{x\to-\infty} \frac{x^3 \cos x}{2+x^2}}_{\to[-1,1]\cdot\infty} + \underbrace{\lim_{x\to-\infty} \frac{1.5^x}{2+x^2}}_{0}$$

$$\Rightarrow \lim_{x\to-\infty} f(x) \text{ oszilliert zwischen } -\infty \text{ und } \infty$$

3.3

```
> fsolve(f,x=-20..-15); # kann man nicht alle auf einmal\
>  suchen ohne zu programmieren?
                                        -17.27914744
```

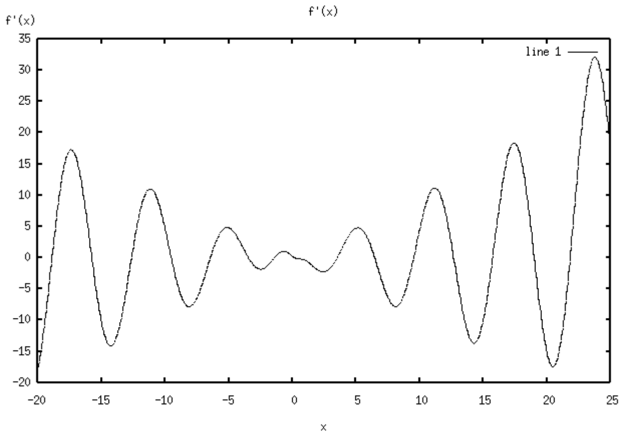
Aus dem Graph der Funktion kann man ablesen, dass es 13 oder 14 Nullstellen geben muss. Durch suchen in weiteren Intervallen findet man folgende 13 Nullstellen:
-17.27914744
-14.13645783
-10.99708682
-7.849761028
-4.732642621
2.075250069
4.626169720
7.907978386
10.92960813
14.24973935
17.07298153
20.98723434
22.59175091

3.4

$$\begin{aligned}
\frac{d}{dx}\left(\frac{f}{g}\right) &= \frac{f'g - fg'}{g^2} \\
f &= 2 + x^3 \cos x + 1.5^x \Rightarrow f' = 3x^2 \cos x - x^3 \sin x + 1.5^x \ln 1.5 \\
g &= 2 + x^2 \Rightarrow g' = 2x \\
\frac{f'g - fg'}{g^2} &=
\end{aligned}$$

$$\frac{(3x^2 \cos x - x^3 \sin x + 1.5^x \ln 1.5)(2 + x^2) - (2 + x^3 \cos x + 1.5^x)2x}{(2 + x^2)^2} =$$

$$\frac{6x^2 \cos x - x^5 \sin x + x^4 \cos x - 2x^3 \sin x - 4x + 1.5^x(\ln 2.25 + \ln 1.5x^2 - 2x)}{(2 + x^2)^2}$$



4

4.1

$$\lim_{x\to 0}\frac{\tan x}{x} \overset{\text{B d' H}}{=} \lim_{x\to 0}\frac{1}{\cos^2(x)\cdot 1} = 1$$

oder Taylor Reihe für $\tan x = x + \ldots \Rightarrow \frac{x}{x} = 1$

4.2

$$\frac{f(a+h)-f(a-h)}{2h} \quad \text{da} \quad (a+h)-(a-h) = 2h$$
$$\frac{f(a+2h)-f(a-h)}{3h} \quad \text{da} \quad (a+2h)-(a-h) = 3h$$

mit Octave

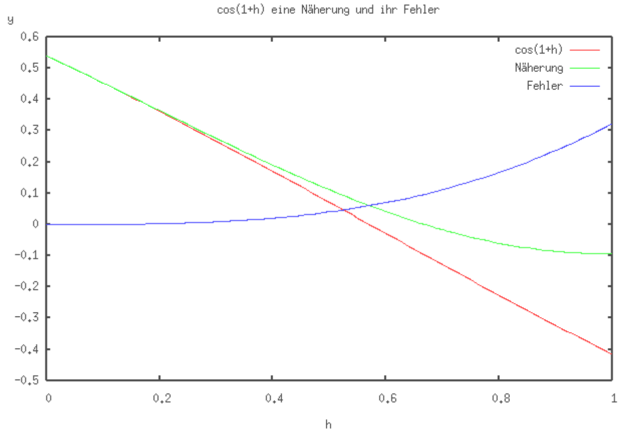
```
> a = 1
a = 1
> i = (1:8)
i =
  1  2  3  4  5  6  7  8

> h = 10.^-i
h =
```



```
   1.0000e-01   1.0000e-02   1.0000e-03   1.0000e-04
   1.0000e-05   1.0000e-06   1.0000e-07   1.0000e-08

> y_r = cos(a) # echter Wert
y_r = 0.54030

> y_0 = (sin(a.+h)-sin(a.-h))./h # müsste um Faktor 2 falsch
sein
y_0 =
  1.0788  1.0806  1.0806  1.0806  1.0806  1.0806  1.0806
  1.0806

> y_1 = (sin(a.+h)-sin(a.-h))./(2.*h)
y_1 =
   0.53940  0.54029  0.54030  0.54030  0.54030  0.54030
   0.54030  0.54030

> y_2 = (sin(a.+h)-sin(h))./h # muss sinnlos sein
y_2 =
   7.9137e+00   8.3683e+01   8.4101e+02   8.4143e+03
   8.4147e+04   8.4147e+05   8.4147e+06   8.4147e+07

> y_3 = (sin(a.+2.*h)-sin(a.-h))./(3.*h)
y_3 =
   0.49571  0.53607  0.53988  0.54026  0.54030  0.54030
   0.54030  0.54030

> y_4 = (sin(a.+h)-sin(a))./h # korrekt Fromel
y_4 =
  0.49736  0.53609  0.53988  0.54026  0.54030  0.54030
  0.54030  0.54030
```

4.3

$$\begin{aligned}
\cos(1+h) &\approx \cos 1 - \sin 1\cdot h + ah^2 \qquad \text{suche } \frac{d}{dh^2} \\
-\sin(1+h) &\approx -\sin 1 + 2ah \\
-\cos(1+h) &\approx 2a \\
-\frac{\cos(1+h)}{2} &\approx a
\end{aligned}$$



5

$$\begin{aligned} \dot{w}(t) &= w(t) - w^2(t) \\ x(t) &:= kw(akt) \quad a, k \in \mathbb{R} \end{aligned}$$

$$\begin{aligned} \dot{x}(t) &= ax(t)\,(k - x(t)) \\ \dot{x}(t) &= akx(t) - ax^2(t) \\ k\dot{w}(akt)ak &= ak^2kw(akt) - ak^2w^2(akt) \\ \dot{w}(akt) &= w(akt) - w^2(akt) \quad \text{mit } t_n = akt \\ \dot{w}(t_n) &= w(t_n) - w^2(t_n) \qquad \blacksquare \end{aligned}$$