UMNW, Mathematik 1, Lösung Serie 4

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1

1.1

```
> g1 := 2*Pi*r+1=2*t+(2*Pi-2*a)*r;
             g1 := 2 Pi r + 1 = 2 t + (2 Pi - 2 a) r

> a := t/(r+h); # sin(a) = a für |a| << 1
                                t
                          a := -----
                               r + h

> g1;
                                /          t  \
              2 Pi r + 1 = 2 t + |2 Pi - 2 -----| r
                                \         r + h/

> t := ((r+h)^2-r^2)^(1/2);
                                 2    2 1/2
                      t := ((r + h)  - r )

> g1;
                       2    2 1/2
2 Pi r + 1 = 2 ((r + h)  - r )

      /                2    2 1/2\
      |         ((r + h)  - r )   |
    + |2 Pi - 2 ------------------| r
      \               r + h       /


> r := 6370*10^3;
                     r := 6370000
```



```
> g1;
                                     2                   1/2
12740000 Pi + 1 = 2 ((6370000 + h)  - 40576900000000)

                                        2                   1/2
                              ((6370000 + h)  - 40576900000000)
+ 12740000 Pi - 12740000 ------------------------------------
                                   6370000 + h

> h_n := fsolve(g1,h); # näherungslösung
                         h_n := 92.68717411

> a := arcsin(t/(r+h)); # ohne näherung für den sinus
                                      2                   1/2
                        ((6370000 + h)  - 40576900000000)
          a := arcsin(------------------------------------)
                                   6370000 + h

> g1;
12740000 Pi + 1 =

                     2                   1/2
    2 ((6370000 + h)  - 40576900000000)    + 12740000 Pi

                                       2                   1/2
                         ((6370000 + h)  - 40576900000000)
    - 12740000 arcsin(------------------------------------)
                                   6370000 + h

> h_g := fsolve(g1,h); # genaue lösung
                          h_g := 121.4543680

> (1-h_n/h_g)*100; # fehler der näherungslösung in %
                              23.68559844
```

1.2

$$\begin{aligned} U &= 2r\pi \\ 2r_{Erde}\pi + \Delta l &= 2(r_{Erde} + h)\pi \\ r_{Erde} + \frac{\Delta l}{2\pi} &= r_{Erde} + h \\ \frac{\Delta l}{2\pi} &= h \\ \frac{1m}{2\pi} &= 0.159m \end{aligned}$$

h hängt nicht vom Radius ab!

2

2.1

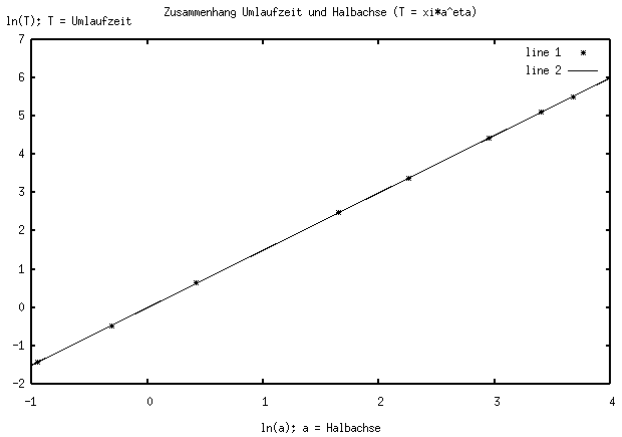
$$\begin{aligned} T &= \xi a^{\eta} \\ \ln T &= \ln \xi + \eta \ln a \end{aligned}$$

Lineare Approximation mit der gleichen Funktion wie in Serie 3

```
> T := [0.241, 0.615, 1.881,\
>  11.862, 29.458, 84.014, 164.739, 247.7];
T := [.241, .615, 1.881, 11.862, 29.458, 84.014,

    164.739, 247.7]

> a := [0.387, 0.732, 1.524,\
>  5.203, 9.539, 19.182, 30.058, 39.44];
a := [.387, .732, 1.524, 5.203, 9.539, 19.182, 30.058,

    39.44]

> read quadapprox;
.
.
.
> qmc := quadapprox(map(ln,a),map(ln,T));
                      "[q, m, c]"

       qmc := [-.004765770, 1.501519469, .9999974987]
```



```
> xi := exp(qmc[1]);
                                xi := .9952455683

> eta := qmc[2];
                               eta := 1.501519469
```



3

```
> f := N -> expand(a*N*(1-N));
            f := N -> expand(a N (1 - N))

> f1(N) := f(N);
                                    2
                  f1(N) := a N - a N

> f2(N) := f(f1(N));
              2      3  2      3  3    2  2    3  4
    f2(N) := a  N - a  N  + 2 a  N  - a  N  - a  N

> f3(N) := f(f2(N));
          5  4      6  6    7  8      7  7    3  2
f3(N) := -a  N  - 2 a  N  - a  N  + 4 a  N  - a  N

        3      4  2      4  3    4  4    5  2
```

```
     + a  N - a  N  + 2 a  N  - a  N  - a  N

          6  3      6  4      5  3      6  5    7  4
     + 2 a  N  - 6 a  N  + 2 a  N  + 6 a  N  - a  N

          7  5      7  6
     + 4 a  N  - 6 a  N

> f4(N) := f(f3(N));
               15  11    7  2      9  3    15  8
f4(N) := 56 a   N   - a  N  + 2 a  N  - a   N

          15  9       15  10    4      15  16
     + 8 a   N  - 28 a   N   + a  N - a   N

          15  15       15  12       15  13
     + 8 a   N   - 70 a   N   + 56 a   N

           15  14    11  4      12  5      13  6
     - 28 a   N   - a   N  + 4 a   N  - 6 a   N

          14  7        14  11      8  4       9  5
     + 4 a   N  + 140 a   N   - 7 a  N  + 10 a  N

        6  2      7  3      8  3      9  4       10  5
     - a  N  + 4 a  N  + 2 a  N  - 7 a  N  + 24 a   N

           11  6       14  8       14  9        14  10
     - 36 a   N  - 28 a   N  + 84 a   N  - 140 a   N

        5  2      6  3      7  4      7  5      7  6
     - a  N  + 2 a  N  - 7 a  N  + 6 a  N  - 2 a  N

          10  8      11  10      12  12       12  11
     - 6 a   N  - 6 a   N   - 2 a   N   + 12 a   N

          8  6       8  5      9  6      9  7    9  8
     - 8 a  N  + 10 a  N  - 8 a  N  + 4 a  N  - a  N

           10  6       11  7       11  8       10  7
     - 36 a   N  + 64 a   N  - 61 a   N  + 24 a   N

           11  9       12  8       12  9       12  10
     + 30 a   N  - 70 a   N  + 60 a   N  - 34 a   N
```



```
           13  10       13  11      13  12
     - 90 a   N   + 36 a   N   - 6 a   N

           14  12       14  13      14  14    4  2
     - 84 a   N   + 28 a   N   - 4 a   N   - a  N

          5  3    5  4    6  4      8  7    8  8
     + 2 a  N  - a  N  - a  N  + 4 a  N  - a  N

          10  4       11  5       12  6       12  7
     - 6 a   N  + 10 a   N  - 22 a   N  + 52 a   N

          13  7       13  8       13  9
     + 36 a   N  - 90 a   N  + 120 a   N

>
```

Die Funktion wird schnell sehr mühsam und scheint nicht vereinfachbar zu sein $\Rightarrow$ keine Fromel für die Funktion $f_n(N)$.