

# UMNW, Mathematik 1, Lösung Serie 4

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1.1

```
> g1 := 2*Pi*r+1=2*t+(2*Pi-2*a)*r;
      g1 := 2 Pi r + 1 = 2 t + (2 Pi - 2 a) r
```

```
> a := t/(r+h); # sin(a) = a für |a| << 1
```

$$a := \frac{t}{r+h}$$

```
> g1;
```

$$2 \text{ Pi } r + 1 = 2 t + \frac{2 \text{ Pi } - 2 \frac{t}{r+h}}{\sqrt{\frac{t}{r+h}}}$$

```
> t := ((r+h)^2-r^2)^(1/2);
```

$$t := ((r+h)^2 - r^2)^{1/2}$$

```
> g1;
```

$$2 \text{ Pi } r + 1 = 2 ((r+h)^2 - r^2)^{1/2}$$

$$\frac{\sqrt{((r+h)^2 - r^2)^{1/2}}}{1 + \frac{2 \text{ Pi } - 2 \frac{((r+h)^2 - r^2)^{1/2}}{r+h}}{\sqrt{((r+h)^2 - r^2)^{1/2}}}}$$

```
> r := 6370*10^3;
```

$$r := 6370000$$

1

```
> g1;
```

$$12740000 \text{ Pi} + 1 = 2 ((6370000 + h)^2 - 4057690000000000)^{1/2}$$

$$+ 12740000 \text{ Pi} - 12740000 \frac{((6370000 + h)^2 - 4057690000000000)^{1/2}}{6370000 + h}$$

```
> h_n := fsolve(g1,h); # näherungslösung
```

$$h_n := 92.68717411$$

```
> a := arcsin(t/(r+h)); # ohne näherung für den sinus
```

$$a := \arcsin\left(\frac{((6370000 + h)^2 - 4057690000000000)^{1/2}}{6370000 + h}\right)$$

```
> g1;
```

$$12740000 \text{ Pi} + 1 =$$

$$2 ((6370000 + h)^2 - 4057690000000000)^{1/2} + 12740000 \text{ Pi}$$

$$- 12740000 \arcsin\left(\frac{((6370000 + h)^2 - 4057690000000000)^{1/2}}{6370000 + h}\right)$$

```
> h_g := fsolve(g1,h); # genaue lösung
```

$$h_g := 121.4543680$$

```
> (1-h_n/h_g)*100; # fehler der näherungslösung in %
```

$$23.68559844$$

## 1.2

$$\begin{aligned}
 U &= 2r\pi \\
 2r_{Erde}\pi + \Delta l &= 2(r_{Erde} + h)\pi \\
 r_{Erde} + \frac{\Delta l}{2\pi} &= r_{Erde} + h \\
 \frac{\Delta l}{2\pi} &= h \\
 \frac{1m}{2\pi} &= 0.159m
 \end{aligned}$$

$h$  hängt nicht vom Radius ab!

## 2

## 2.1

$$\begin{aligned}
 T &= \xi a^\eta \\
 \ln T &= \ln \xi + \eta \ln a
 \end{aligned}$$

Lineare Approximation mit der gleichen Funktion wie in Serie 3

```

> T := [0.241, 0.615, 1.881, \
> 11.862, 29.458, 84.014, 164.739, 247.7];
T := [.241, .615, 1.881, 11.862, 29.458, 84.014,
      164.739, 247.7]

> a := [0.387, 0.732, 1.524, \
> 5.203, 9.539, 19.182, 30.058, 39.44];
a := [.387, .732, 1.524, 5.203, 9.539, 19.182, 30.058,
      39.44]

```

```

> read quadapprox;
.
.
.
> qmc := quadapprox(map(ln,a),map(ln,T));
      "[q, m, c]"

qmc := [-.004765770, 1.501519469, .9999974987]

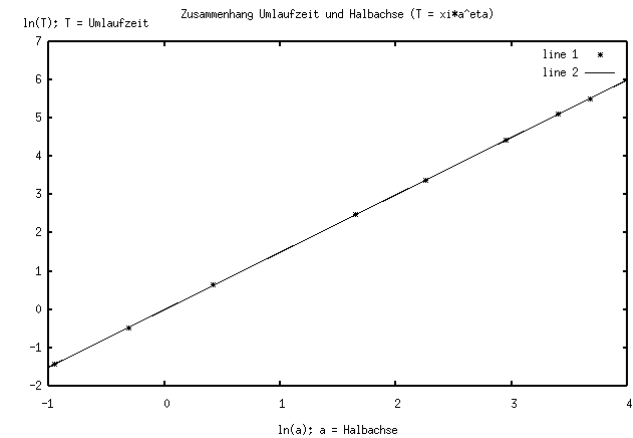
```

```

> xi := exp(qmc[1]);
      xi := .9952455683

> eta := qmc[2];
      eta := 1.501519469

```



## 3

```

> f := N -> expand(a*N*(1-N));
      f := N -> expand(a N (1 - N))

> f1(N) := f(N);
      f1(N) := a N^2 - a N

> f2(N) := f(f1(N));
      f2(N) := a^2 N^4 - 2 a^3 N^3 + 2 a^3 N^2 - a^4 N

> f3(N) := f(f2(N));
      f3(N) := -a^5 N^6 + 2 a^6 N^5 - a^7 N^4 + 4 a^7 N^3 - a^8 N^2

```

```

+ a N - a N + 2 a N - a N - a N
      6 3      6 4      5 3      6 5      7 4
+ 2 a N - 6 a N + 2 a N + 6 a N - a N
      7 5      7 6
+ 4 a N - 6 a N

> f4(N) := f(f3(N));
      15 11      7 2      9 3      15 8
f4(N) := 56 a N - a N + 2 a N - a N
      15 9      15 10      4      15 16
+ 8 a N - 28 a N + a N - a N
      15 15      15 12      15 13
+ 8 a N - 70 a N + 56 a N
      15 14      11 4      12 5      13 6
- 28 a N - a N + 4 a N - 6 a N
      14 7      14 11      8 4      9 5
+ 4 a N + 140 a N - 7 a N + 10 a N
      6 2      7 3      8 3      9 4      10 5
- a N + 4 a N + 2 a N - 7 a N + 24 a N
      11 6      14 8      14 9      14 10
- 36 a N - 28 a N + 84 a N - 140 a N
      5 2      6 3      7 4      7 5      7 6
- a N + 2 a N - 7 a N + 6 a N - 2 a N
      10 8      11 10      12 12      12 11
- 6 a N - 6 a N - 2 a N + 12 a N
      8 6      8 5      9 6      9 7      9 8
- 8 a N + 10 a N - 8 a N + 4 a N - a N
      10 6      11 7      11 8      10 7
- 36 a N + 64 a N - 61 a N + 24 a N
      11 9      12 8      12 9      12 10
+ 30 a N - 70 a N + 60 a N - 34 a N

```

```

      13 10      13 11      13 12
- 90 a N + 36 a N - 6 a N
      14 12      14 13      14 14      4 2
- 84 a N + 28 a N - 4 a N - a N
      5 3      5 4      6 4      8 7      8 8
+ 2 a N - a N - a N + 4 a N - a N
      10 4      11 5      12 6      12 7
- 6 a N + 10 a N - 22 a N + 52 a N
      13 7      13 8      13 9
+ 36 a N - 90 a N + 120 a N

```

&gt;

Die Funktion wird schnell sehr mühsam und scheint nicht vereinfachbar zu sein  $\Rightarrow$  keine Formel für die Funktion  $f_n(N)$ .