UMNW, Mathematik 1, Lösung Serie 4

Thomas Kuster

18. November 2003

1

1.1

```
> g1 := 2*Pi*r+1=2*t+(2*Pi-2*a)*r;
              g1 := 2 Pi r + 1 = 2 t + (2 Pi - 2 a) r

> a := t/(r+h); # sin(a) = a für |a| << 1
                                  t
                             a := -----
                                  r + h

> g1;
                                  /           t  \
               2 Pi r + 1 = 2 t + |2 Pi - 2 -----| r
                                  \         r + h/

> t := ((r+h)^2-r^2)^(1/2);
                                   2    2 1/2
                      t := ((r + h)  - r )

> g1;
                       2    2 1/2
2 Pi r + 1 = 2 ((r + h)  - r )

      /                 2    2 1/2\
      |         ((r + h)  - r )   |
    + |2 Pi - 2 ------------------| r
      \               r + h       /


> r := 6370*10^3;
                     r := 6370000
```

```
> g1;
                                     2                    1/2
12740000 Pi + 1 = 2 ((6370000 + h)  - 40576900000000)

                                           2                    1/2
                         ((6370000 + h)  - 40576900000000)
+ 12740000 Pi - 12740000 ------------------------------------
                                      6370000 + h

> h_n := fsolve(g1,h); # näherungslösung
                        h_n := 92.68717411

> a := arcsin(t/(r+h)); # ohne näherung für den sinus
                                    2                    1/2
                     ((6370000 + h)  - 40576900000000)
         a := arcsin(------------------------------------)
                                 6370000 + h

> g1;
12740000 Pi + 1 =

                    2                    1/2
    2 ((6370000 + h)  - 40576900000000)     + 12740000 Pi

                                     2                    1/2
                       ((6370000 + h)  - 40576900000000)
    - 12740000 arcsin(------------------------------------)
                                   6370000 + h

> h_g := fsolve(g1,h); # genaue lösung
                        h_g := 121.4543680

> (1-h_n/h_g)*100; # fehler der näherungslösung in %
                            23.68559844
```

1.2

$$
\begin{aligned}
U &= 2r\pi \\
2r_{Erde}\pi + \Delta l &= 2(r_{Erde} + h)\pi \\
r_{Erde} + \frac{\Delta l}{2\pi} &= r_{Erde} + h \\
\frac{\Delta l}{2\pi} &= h \\
\frac{1m}{2\pi} &= 0.159m
\end{aligned}
$$

h hängt nicht vom Radius ab!

2

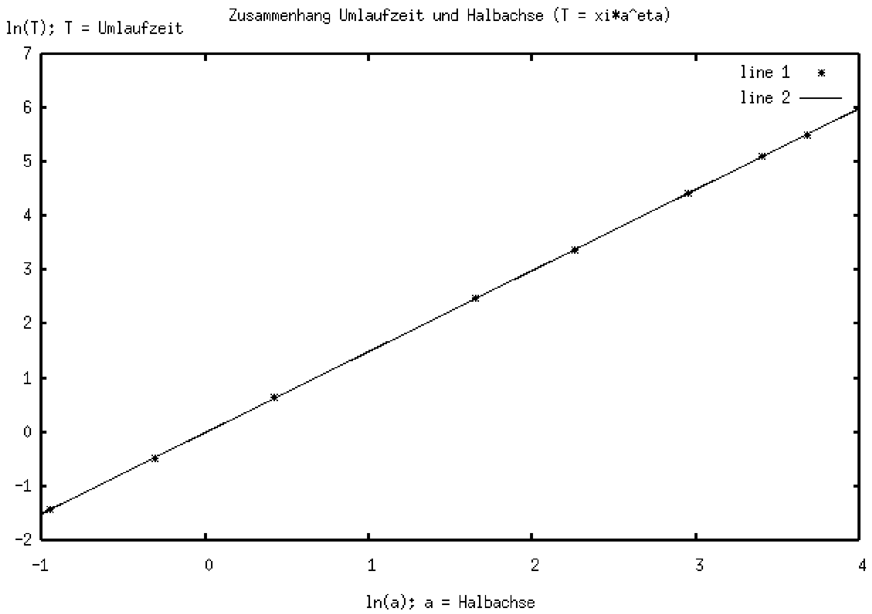
2.1

$$
\begin{aligned}
T &= \xi a^{\eta} \\
\ln T &= \ln \xi + \eta \ln a
\end{aligned}
$$

Lineare Approximation mit der gleichen Funktion wie in Serie 3

```
> T := [0.241, 0.615, 1.881,\
>  11.862, 29.458, 84.014, 164.739, 247.7];
T := [.241, .615, 1.881, 11.862, 29.458, 84.014,

    164.739, 247.7]

> a := [0.387, 0.732, 1.524,\
>  5.203, 9.539, 19.182, 30.058, 39.44];
a := [.387, .732, 1.524, 5.203, 9.539, 19.182, 30.058,

    39.44]

> read quadapprox;
.
.
.
> qmc := quadapprox(map(ln,a),map(ln,T));
                     "[q, m, c]"

       qmc := [-.004765770, 1.501519469, .9999974987]
```

```
> xi := exp(qmc[1]);
                                xi := .9952455683

> eta := qmc[2];
                               eta := 1.501519469
```



3

```
> f := N -> expand(a*N*(1-N));
             f := N -> expand(a N (1 - N))

> f1(N) := f(N);
                                         2
                   f1(N) := a N - a N

> f2(N) := f(f1(N));
              2       3  2       3  3    2  2    3  4
    f2(N) := a  N - a  N  + 2 a  N  - a  N  - a  N

> f3(N) := f(f2(N));
           5  4      6  6    7  8      7  7    3  2
f3(N) := -a  N  - 2 a  N  - a  N  + 4 a  N  - a  N

        3      4  2      4  3    4  4    5  2
```

$$+ a N - a N + 2 a N - a N - a N$$

$$+ 2 a^{6} N^{3} - 6 a^{6} N^{4} + 2 a^{5} N^{3} + 6 a^{6} N^{5} - a^{7} N^{4}$$

$$+ 4 a^{7} N^{5} - 6 a^{7} N^{6}$$

```
> f4(N) := f(f3(N));
```

$$f4(N) := 56 a^{15} N^{11} - a^{7} N^{2} + 2 a^{9} N^{3} - a^{15} N^{8}$$

$$+ 8 a^{15} N^{9} - 28 a^{15} N^{10} + a^{4} N - a^{15} N^{16}$$

$$+ 8 a^{15} N^{15} - 70 a^{15} N^{12} + 56 a^{15} N^{13}$$

$$- 28 a^{15} N^{14} - a^{11} N^{4} + 4 a^{12} N^{5} - 6 a^{13} N^{6}$$

$$+ 4 a^{14} N^{7} + 140 a^{14} N^{11} - 7 a^{8} N^{4} + 10 a^{9} N^{5}$$

$$- a^{6} N^{2} + 4 a^{7} N^{3} + 2 a^{8} N^{3} - 7 a^{9} N^{4} + 24 a^{10} N^{5}$$

$$- 36 a^{11} N^{6} - 28 a^{14} N^{8} + 84 a^{14} N^{9} - 140 a^{14} N^{10}$$

$$- a^{5} N^{2} + 2 a^{6} N^{3} - 7 a^{7} N^{4} + 6 a^{7} N^{5} - 2 a^{7} N^{6}$$

$$- 6 a^{10} N^{8} - 6 a^{11} N^{10} - 2 a^{12} N^{12} + 12 a^{12} N^{11}$$

$$- 8 a^{8} N^{6} + 10 a^{8} N^{5} - 8 a^{9} N^{6} + 4 a^{9} N^{7} - a^{9} N^{8}$$

$$- 36 a^{10} N^{6} + 64 a^{11} N^{7} - 61 a^{11} N^{8} + 24 a^{10} N^{7}$$

$$+ 30 a^{11} N^{9} - 70 a^{12} N^{8} + 60 a^{12} N^{9} - 34 a^{12} N^{10}$$

$$- 90\, a^{13} N^{10} + 36\, a^{13} N^{11} - 6\, a^{13} N^{12}$$

$$- 84\, a^{14} N^{12} + 28\, a^{14} N^{13} - 4\, a^{14} N^{14} - a^{4} N^{2}$$

$$+ 2\, a^{5} N^{3} - a^{5} N^{4} - a^{6} N^{4} + 4\, a^{8} N^{7} - a^{8} N^{8}$$

$$- 6\, a^{10} N^{4} + 10\, a^{11} N^{5} - 22\, a^{12} N^{6} + 52\, a^{12} N^{7}$$

$$+ 36\, a^{13} N^{7} - 90\, a^{13} N^{8} + 120\, a^{13} N^{9}$$

```
>
```

Die Funktion wird schnell sehr mühsam und scheint nicht vereinfachbar zu sein $\Rightarrow$ keine Fromel für die Funktion $f_n(N)$.