UMNW, Mathematik 1, Lösung Serie 1

Thomas Kuster

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1

$$
\begin{aligned}
0.001m \cdot 2^x &= 150 \cdot 10^9 m \\
2^x &= \frac{150 \cdot 10^9 m}{0.001m} \\
x \cdot \ln 2 &= \ln \frac{150 \cdot 10^9}{0.001} \\
x &= \frac{\ln 1.5 \cdot 10^{14}}{\ln 2} \\
x &\approx 47.09 \\
&\Rightarrow \text{48 mal Falten (aufrunden)}
\end{aligned}
$$

2

$$
\begin{aligned}
N_0 \cdot \lambda^k &= 2N_0 \\
\lambda^k &= 2 \\
k \ln \lambda &= \ln 2 \\
k &= \frac{\ln 2}{\ln \lambda}
\end{aligned}
$$

Lösen mit Octave (http://www.octave.org)

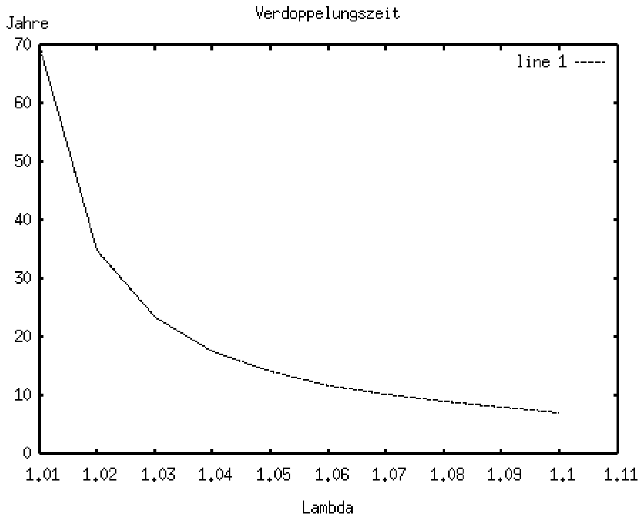
```
octave:1> lambda = (1.01:0.01:1.1)
lambda =
1.01 1.02 1.03 1.04 1.05 1.06 1.07 1.08 1.09 1.10

octave:2> log(2)./log(lambda)
69.6607
35.0028
23.4498
17.6730
14.2067
```



```
11.8957
10.2448
 9.0065
 8.0432
 7.2725

octave:3> title('Verdoppelungszeit');
octave:4> xlabel('Lambda');
octave:5> ylabel('Jahre');
octave:6> plot(lambda,log(2)./log(lambda))
```



3

3.1

$$
\begin{aligned}
48503000 \cdot 1.025^{2005-1985} &= x \\
79477813 &\approx x
\end{aligned}
$$

3.2

$$\begin{aligned}\left((48503000 \cdot 1.025^{10})1.02^5\right)1.015^5 &= x\\ 73847929 &\approx x\end{aligned}$$

3.3

$$\begin{aligned}\left((48503000 \cdot 1.025^{10})1.03^5\right)1.035^5 &= x\\ 85486025 &\approx x\end{aligned}$$

3.4

$$\begin{aligned}N_0 \cdot p &= N_k\\ p &= \frac{N_k}{N_0}\end{aligned}$$

$$\begin{aligned}p_1 &= \frac{79477813}{48503000} \Rightarrow 63.9\%\\ p_2 &= \frac{73847929}{48503000} \Rightarrow 52.3\%\\ p_3 &= \frac{85486025}{48503000} \Rightarrow 76.2\%\end{aligned}$$

4

4.1

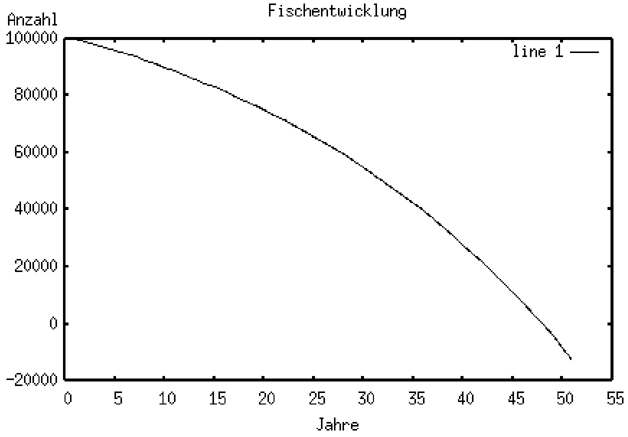
$$\begin{aligned}N_0\lambda^k + \sum_{i=0}^{k-1} m\lambda^i &= x\\ 8 \cdot 10^7 \cdot 0.99^k + \sum_{i=0}^{k-1} 3 \cdot 10^5 \cdot 0.99^i &= x\end{aligned}$$

```
octave:1> 8*10^7 *0.99^10 + sum(3*10^5*0.99.^(0:(10-1)))
ans =  7.5219e+07
```


4.2

$$\begin{aligned}N_0\lambda^k + \sum_{i=0}^{k-1} m\lambda^i &= x\\ 10^5 \cdot 1.03^k + \sum_{i=0}^{k-1} -4 \cdot 10^3 \cdot 1.03^i &= x\end{aligned}$$

```
octave:1> p = 100000
p = 100000
octave:2> for i=1:50
> p = [p 10^5*1.03^i - 4*10^3*sum(1.03.^(0:(i-1)))];
> end
octave:3> title('Fischentwicklung');
octave:4> xlabel('Jahre');
octave:5> ylabel('Anzahl');
octave:6> plot(p)
```



oder

$$\begin{aligned}
N_0\lambda^k + \sum_{i=0}^{k-1} m\lambda^i &= x \\
N_0\lambda^k + m\frac{\lambda^k - 1}{\lambda - 1} &= x \\
10^5 \cdot 1.03^k - 4 \cdot 10^3 \frac{1.03^k - 1}{1.03 - 1} &= x
\end{aligned}$$

```
octave:1> k = (1:50);
octave:2> N = 10^5*1.03.^k + (-4*10^3)*(1.03.^k-1)./(1.03-1)
octave:3> plot(k,N);
```

Wann hat es keine Fische mehr?

$$\begin{aligned}
N_0\lambda^k + m\frac{\lambda^k - 1}{\lambda - 1} &\stackrel{!}{=} 0 \\
m\frac{\lambda^k - 1}{\lambda - 1} &= -N_0\lambda^k \\
\lambda^k - 1 &= \frac{(-N_0\lambda^k)(\lambda - 1)}{m} \\
-1 &= \frac{(-N_0\lambda^k)(\lambda - 1)}{m} - \lambda^k \\
1 &= \left(\frac{N_0(\lambda - 1)}{m} + 1\right)\lambda^k \\
1 &= \frac{N_0\lambda - N_0 + m}{m}\lambda^k \\
\frac{m}{N_0\lambda - N_0 + m} &= \lambda^k \\
\ln\frac{m}{N_0\lambda - N_0 + m} &= k\ln\lambda \\
\frac{\ln\frac{m}{N_0\lambda - N_0 + m}}{\ln\lambda} &= k \\
\frac{\ln 4}{\ln 1.03} &\approx 46.9\text{Jahre}
\end{aligned}$$